Course: TOPICS in GAME THEORY

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Schedule: Rogla Summer School; July 22 - 28, 2017.

No Prerequisites. Knowledge of basic concepts of Linear Programming and Graph Theory would be useful, but everything will be explained in class.

Literature: There is no single textbook for this course; instead, there are references to very many books and papers. It is not necessary to get (download and read) all of them. In fact, one or two (available freely in internet) will be enough for each topic. Note that RUTCOR Research Reports (RRR) are available online.

Language: English.

Nine topics of the course are listed below. Not all of them will be covered. The first three will be covered for sure, plus 2-3 among the others; see the slides.

1. Matrix games.

a) max min, min max, and saddle point;
b) solution in pure and mixed strategies;
c) Solving matrix games in mixed strategies and strong duality theorem in Linear Programming; Matrix Game Theorem, von Neumann (1928).

Thomas S. Ferguson; Game Theory; Class notes for Math 167, Fall 2000; open access at
http://www.cs.cmu.edu/afs/cs/academic/class/15859-f01/www/notes/mat.pdf

Guillermo Owen; Game Theory; Business & Economics, Academic Press (1995) 447 pages; first edition in 1968; in Russian: Moscow, Mir; 1971.

John von Neumann and Oscar Morgenstern, Theory of games and economic behavior, Princeton University Press, Princeton, New Jersey, 1944; translated into Russian, Moscow, Nauka; 1970.

2. Bimatrix games and n-person games in normal form.

a) Brouwer (1911) and Kakutani's (1941) Fixed Point Theorems.
b) Best response and Nash equilibrium (NE).
c) Nash theorem (1950): each finite n-person games in the normal form has a NE in mixed strategies. In 1950 Nash gave two different proofs deriving it from the Kakutani and Brouwer Theorems.

Thomas S. Ferguson; Game Theory; Class notes for Math 167, Fall 2000; open access at
http://www.cs.cmu.edu/afs/cs/academic/class/15859-f01/www/notes/mat.pdf

Guillermo Owen; Game Theory; Business & Economics, Academic Press (1995) 447 pages; first edition in 1968; in Russian: Moscow, Mir; 1971.

John Forbs Nash Jr.; Equilibrium points in n-person games; Proceedings of the National Academy of Sciences, USA 36 (1950) 48–49.

John Forbs Nash Jr.; Non-cooperative games; Doctoral dissertation, 1950, Princeton University.

3. Nash equilibrium (NE) in pure strategies for (two-person) games in normal form.

a) Game Forms (GFs) and Effectivity Functions (EFFs); an introduction.

b) Tight game forms; four equivalent definitions of tightness.

c) Further equivalent properties of two-person game forms: tightness, ±1- , zero-sum-, and NE-solvability; the Bottle Neck Extrema Theorem, Fulkerson-Edmonds (1970).

d) An extension to infinite two-person game forms.

e) Axiom of Choice (Ernst Zermelo, 1904) versus Axiom of Determinacy (Jan Mycielski and Hugo Steinhaus, 1962).

f) No extension to 3-person game forms; examples.

g) Applications of tightness for positional and cycle games and in veto-voting.

Herve Moulin; The strategy of social choice; Advanced textbooks in Economics, vol. 18, North Holland, 1983.

Herve Moulin; Game Theory for the Social Sciences (Studies in Game Theory and Mathematical Economics); NYU Press, 1986.

Bezalel Peleg; Game theoretic analysis of voting in committees; Econometric Society Publication, volume 7, Cambridge Univ. Press; Cambridge, London, New York, New Rochelle, Melbourne, Sydney; 1984.

Jack Edmonds and Donald R. Fulkerson; Bottleneck Extrema; RM-5375-PR, The Rand Corporation, Santa Monica, Ca., Jan. 1968; J. Combin. Theory 8 (1970) 299–306.

Vladimir Gurvich; To theory of multi-step games; J. Vychisl. matem. i matem. fiz., 13:6 (1973) 1485-1500; in Russian, Englisj translation in USSR Comput. Math. and Math. Phys. 13:6 (1973) 143-161.

Vladimir Gurvich; Solution of positional games in pure strategies; J. Vychisl. matem. i matem. fiz., 15:2 (1975) 358-371; in Russian, English translation in USSR Comput. Math. and Math. Phys. 15:2 (1975) 74-87.

V. Gurvich, Equilibrium in pure strategies, Soviet Mathematics Doklady 303:4 (1988) 538–542; in Russian, English transl. in Soviet Math. Dokl. 38:3 (1989) 597–602.

Vladimir Gurvich; War and peace in veto voting; European J. of Operations Research, 185 (2008) 438-443; preprints in open access: RUTCOR Research Report 22-2005, Rutgers University.

Andrei I. Gol'berg and Vladimir A. Gurvich; Some properties of tight cyclic game forms, Doklady Akad. Nauk. 318:6 (1991), 1289-1294 (in Russian); English translation in: Russian Acad. Sci. Dokl. Math., 43:3 (1991) 898-903.

Andrei I. Gol'berg and Vladimir A. Gurvich; A tightness criterion for reciprocal bipartite cyclic game forms, Doklady Akad. Nauk. 323:3 (1992), 398-405 (in Russian); English translation in: Russian Acad. Sci. Dokl. Math., 45:2 (1992) 348-354.

Endre Boros, Vladimir Gurvich, Kazuhisa Makino, and Wei Shao; Nash-solvabile two-person symmetric cycle game forms; Discrete Applied Math. 159:15 (2011) 1461–1487; preprints in open access: Nash-solvability of bidirected cyclic game forms; RUTCOR Research Report RRR-30-2007, revised in RRR-20-2009, Rutgers University.

Vladimir Gurvich and Vladimir Oudalov; A four-person chess-like game without Nash equilibria in pure stationary strategies, Business Informatics 1:31 (2015) 68–76; preprint in open access at
arXiv 1411.0349 at http://arxiv.org/abs/1411.0349

Endre Boros, V. Gurvich, Martin Milanic, Vladimir Oudalov, and Jernej Vicic; A three-person chess-like game without Nash equilibria; submitted to Discrete Applied Mathematics, October 16, 20016; now available at: https://scirate.com/arxiv/1610.07701

Frank Tuijsman and Tilrukkannamangal E.S. Raghavan; Perfect Information Stochastic Games and Related classes Int. J. Game Theory (IJGT) 26 (1997) 403–408.

4. On minimal and locally minimal NE-free bimatrix games.

a) Minimal saddle point free matrix games.
b) Locally minimal NE-free bimatrix games.

L. S. Shapley, Some topics in two-person games, in : "Advances in Game Theory" (M. Drescher, L.S. Shapley, and A.W. Tucker, eds.), Annals of Mathematical Studies, AM52, Princeton University Press (1964) 1-28.

Nikolay S. Kukushkin, Shapley's 2×2 theorem for game forms; Economics Bullletin, in open access at
http://economicsbulletin.vanderbilt.edu/2007/volume3/EB-07C70017A.pdf;
see also Technical Report, Department of Mathematical Methods for Economic Decision Analysis, Computing Center of Russian Academy of Sciences;
http://www.ccas.ru/mmes/mmeda/ququ/Shapley.pdf

Endre Boros, Khaled Elbassioni, Vladimir Gurvich, Kazuhisa Makino, and Vladimir Oudalov; Complete characterization of Nash-solvabile bimatrix games in terms of excluding certain 2×2 subgames, Lecture Notes in Computer Science 5010 (2008) 99-109; (E. Hirsh, A. Razborov, A. Semenov, and A. Slissenko eds.); Computer Science in Russia (CSR-08), Moscow, June 7-11, 2008.

Endre Boros, Khaled Elbassioni, Vladimir Gurvich, Kazuhisa Makino, and Vladimir Oudalov; Sufficient conditions for the existence of Nash equilibria in

bimatrix games in terms of forbidden 2×2 subgames, International Journal of Game Theory, DOI 10.1007/s00182-015-0513-7; in open access at RRR-05-2014.

E. Boros, V. Gurvich, and K. Makino; Minimal and locally minimal games and game forms; Discrete Mathematics 309:13 (2009) 4456–4468; preprint in open access: RUTCOR Research Report, RRR-28-2007, Rutgers University.

5. Domination of strategies and dominance equilibrium (DE).

a) Domination of strategies in acyclic positional games and for the games in normal form; uniqueness of DE; every DE is a NE.

b) DE in acyclic positional games and in veto-voting.

Herve Moulin; The strategy of social choice; Advanced textbooks in Economics, vol. 18, North-Holland, 1983.

Herve Moulin; Game Theory for the Social Sciences (Studies in Game Theory and Mathematical Economics); NYU Press, 1986.

Herve Moulin; Dominance-solvable voting schemes; Econometrica 47 (1979) 1337-1351.

Harold W. Kuhn; Extensive games and the problem of information 193–216; in Contributions to the theory of games, v.II; Annals of Math. Studies 28 (Harold W. Kuhn and A. W Tucker, eds.) Princeton University Press, 1953.

David Gale; A theory of N-person games with perfect information; Proceedings of National Academy of Sciences 39 (1953) 496–501.

6. On Acyclicity, Nash- and Dominance-solvability of Two-Person Game Forms.

a) Tight and Totally Tight Game Forms.

b) Total tightness and Acyclicity are equivalent.
(Tightness and Nash-solvability are equivalent.)

c) Acyclicity implies Dominance-solvability implies Nash-solvability.

d) On extensions to game forms of several players.

Endre Boros, Vladimir Gurvich, Kazuhisa Makino, and David Papp; On acyclic, or totally tight, two-person game forms; Discrete Mathematics, 310:6-7 (2010) 1135 – 1151; in open access at RUTCOR Research Report, RRR-03-2008.

Endre Boros, Ondrej Cepek, Vladimir Gurvich, Kazuhisa Makino, Igor E. Zverovich. Separable discrete functions; in open access at RUTCOR Research Report, RRR-26-2009, Rutgers University.

Endre Boros, Ondrej Cepek, and Vladimir Gurvich. Total tightness implies Nash-solvability for three-person game forms; Discrete Mathematics 312:8 (2012) 1436–1443, preprint in open access at Three-person totally tight game forms are Nash-solvable; RUTCOR Research Report, RRR-11-2011.

D. Anderson, V. Gurvich, and T. Hansen; On acyclicity of games with cycles; Discrete Applied Math. 158:10 (2010) 1049-1063; in open access at RUTCOR Research Report, RRR-18-2008; Rutgers University.

7. Effectivity Functions in Game, Voting, and Graph Theories.

a) An Effectivity function is a Boolean function whose set of variables consists of both the players (voters) and the outcomes (candidates).

b) Basic properties of EFFs: monotonisity, sub- and super-additivity, duality and selfduality, balanceness and convexity.

c) Playing and playing-minor EFFS; a characterization of EFFs of game forms; Moulin-Peleg's Theorem.

d) Tightness of a game form is equivalent to the selfduality of the corresponding EFF.

e) War and Peace in Veto Voting.

f) Core $C(eff, u)$ and Scarf Theorem.

g) Connections to Graph Theory; Proof of the Berge-Duchet Conjecture; Generalizing Gale-Shapley's Stable Matching Theorem; Perfect Graphs, Kernels and Cores of Cooperative Games.

Herve Moulin; The strategy of social choice; Advanced textbooks in Economics, vol. 18, North Holland, 1983.

Herve Moulin and Bezalel Peleg; Cores of effectivity functions and implementation theory; J. of Math. Economics 10 (1982) 115-145.

Bezalel Peleg; Game theoretic analysis of voting in committees; Econometric Society Publication, volume 7, Cambridge Univ. Press; Cambridge, London, New York, New Rochelle, Melbourne, Sydney; 1984.

Bezalel Peleg, Core stability and duality of effectivity functions, in: "Selected topics in operations research and mathematical economics" (G. Hammer and D. Pallaschke eds.), Springer-Verlag (1984) 272-287.

Bezalel Peleg; Effectivity functions, game forms, games, and rights; Social choice and welfare 15 (1998) 67-80.

David Gale and Lloyd S. Shapley; College Admissions and the Stability of Marriage; American Mathematical Monthly; 69 (1962) 9–14. doi:10.2307/2312726.

Endre Boros and Vladimir Gurvich; Perfect graphs are kernel-solvable; Discrete Mathematics 159 (1996) 35-55; preprint in open access at RUTCOR Research Report, RRR-16-1994; Rutgers University.

Endre Boros and Vladimir Gurvich; Perfect graphs, kernels, and cores of cooperative games; Discrete Mathematics 306:19-20 (2006) 2336-2354; preprint in open access at RUTCOR Research Report, RRR 12-2003, Rutgers University.

Vladimir Gurvich; Some properties of effectivity functions; Doklady Akad. Nauk SSSR, 307:6 (1989) 1311-1317 (in Russian); English transl. in Soviet Math. Dokl. 40 (1) (1990) 244-250.

Vladimir Gurvich; Algebraic properties of effectivity functions; Doklady Acad. Nauk SSSR, 323:1 (1992) 19-24 (in Russian); English translation in Russian Acad. Sci. Dokl. Math. 45 (2) (1992) 245-251.

Vladimir Gurvich; Effectivity functions and informational extensions of game forms and game correspondences; Russian Math. Surveys 47 (1992) 208-210; doi: 10.1070/RM1992v047n06ABEH000971

Vladimir Gurvich; War and peace in veto voting; European J. of Operations Research, 185 (2008) 438-443; preprint in open access at RUTCOR Research Report, RRR-22-2005, Rutgers University.

8. Characterizing Normal Forms of Positional Games and Read-Once Boolean Functions.

a) A game form g is the normal form of a positional structure (modeled by a tree) if and only if g is tight and rectangular.

a) Tight and rectangular game forms, read-once Boolean functions, and Π- and Δ-free complete edge-colored graphs; bijections.

Yves Crama and Peter Hammer eds., Boolean functions: Theory, algorithms, and applications, Cambridge University Press, 2011; Chapter 10, pp. 448-486, Read once Boolean functions; by Martin Golumbic and Vladimir Gurvich.

M. Karchmer, N. Linial, L. Newman, M. Saks, and A. Wigderson; Combinatorial characterization of read-once formulae; Discrete Mathematics 114 (1993) 275-282; in open access at
http : //www.cs.huji.ac.il/ nati/PAPERS/read_once.pdf

Vladimir Gurvich, On repetition-free Boolean functions (O bespovtornyh Bulevyh funkcijah), Uspechi mat. nauk (Russian Math. Surveys) 32:1 (1977) 183-184 (in Russian).

Vladimir Gurvich; Criteria for repetition freeness of functions in the algebra of logic; Doklady Akad. Nauk SSSR 318:3 (1991) 532-537 in Russian, English transl. in Russian Acad. Sci. Dokl. Math. 43:3 (1991) 721-726.

Vladimir Gurvich; Decomposing complete edge-chromatic graphs and hypergraphs, Revisited; Discrete Applied Mathematics 157 (2009) 3069–3085; in open access at RUTCOR Research Report, RRR-29-2006.

Endre Boros and Vladimir Gurvich; Sandwich problem for Π- and Δ-free multigraphs and its applications to positional games; Discrete Mathematics 338:12 (20015) 2421–2436; in open access at RUTCOR Research Report, RRR-11-2013, Rutgers University.

Vladimir Gurvich; On the normal form of positional games, Doklady Akad. Nauk SSSR 264:1 (1982) 30–33; in Russian, English transl. in Soviet Math Dokl. 25:3 (1982) 572–575.

9. Combinatorial and impartial games.

a) Kernels, N- and P-positions.

b) Sprague-Grundy theory; minimal excludant mex.

c) Sum of impartial games, Moore's k-sum and hypergraph-sum.

d) Game of NIM and its modifications: Wythoff's NIM(1), Fraenkel's $NIM(a)$, and two-parametrical $NIM(a, b)$; generalized minimal excludant mex_b.

e) Game Euclid and its extensions.

f) On tame, pet, domestic, miserable and strongly miserable impartial games.

g) Games of no return and game "Geography"; complexity of solution for the cases of directed and undirected graphs.

An introduction to the Sprague-Grundy theory in Russian; open access at $http://e-maxx.ru/algo/sprague\_grundy$

E.R. Berlekamp, J.H. Conway, and R.K. Guy; Winning ways for your mathematical plays; vol.1-4, second edition, A.K. Peters, Natick, MA, 2001 - 2004, first edition, Academic Press, Inc., 1982.

John H, Conway; On Numbers and Games; first edition: Academic Press, New York, 1976; second edition: CRC Press, Dec 11, 2000.

Robert J. Nowakowski ed.; Games of No Chance; Mathematical Sciences Research Institute Publications (MSRI), vol.29, Cambridge Univ. Press, 1998.

Robert J. Nowakowski ed.; More games of No Chance 2; Math. Sciences Research Institute Publications (MSRI), vol.42, Cambridge Univ. Press, 2002.

M.H. Albert and R.J. Nowakowski eds.; Games of No Chance 3; Math. Sci. Research Institute Publications (MSRI), vol.56, Cambridge Univ. Press, 2009.

Charles L. Bouton; Nim, A Game with a Complete Mathematical Theory; The Ann. of Math., 2nd Ser. 3:1/4 (1901 - 1902) 35–39; in open access at https://paradise.caltech.edu/ist4/lectures/Bouton1901.pdf

Willem A. Wythoff; A modification of the game of Nim; Nieuw Arch. Wisk. 7 (1907) 199–202.

Eliakim H. Moore; A generalization of the game called nim; Annals of Math. (2) 11:3 (1910).

Roland P. Sprague; Über mathematische Kampfspiele; Tohoku Math. J. 41 (1935–36) 438–444.

Roland P. Sprague; Über zwei Abarten von Nim; Tohoku Math. J. 43 (1937) 451–454.

Patrick M. Grundy; Mathematics of games; Eureka 2 (1939) 6-8.

Gabbriel Nivasch; The Sprague-Grundy function for Wythoff's game: On the location of the g-values; M. Sc. Thesis, 2004; Weizmann Institute of Sciences, Rehovot 76100, Israel; in open access at http://www.wisdom.weizmann.ac.il/ gabrieln.

Gabbriel Nivasch; More on the Sprague-Grundy function for Wythoff's game; Games of No Chance 3, MSRI Publications, Volume 56, 2009; in open access at http://www.msri.org/people/staff/levy/files/Book56/43nivasch.pdf

Gabbriel Nivasch; Euclid; The SP-function of the game is in open access at http://www.gabrielnivasch.org/fun/combinatorial-games/euclid

A.J. Cole, A.J.T. Davie; A game based on the Euclidean algorithm and a winning strategy for it; Math. Gazette 53 (1969) 354–357.

G. Nivasch; The Sprague-Grundy function of the game Euclid, Discrete Mathematics 306 (2006) 27982800.

A.S. Fraenkel, E.R. Scheinerman and D. Ullman: Undirected edge geography, Theoretical Computer Science 112 (1993) 371-381.

Jack Edmonds and Vladimir Gurvich; Games of no return; manuscript, can be e-mailed by request.

Vladimir Gurvich, On the misere version of game Euclid and miserable games, Discrete Math. 307 (2007), 1199-1204.

Vladimir Gurvich; Miserable and strongly miserable impartial games; preprint in open access at RUTCOR Research Report 18-2011, Rutgers University.

Vladimir Gurvich; On tame, pet, miserable and strongly miserable impartial games; preprint in open access at RUTCOR Research Report, RRR-18-2012, Rutgers University.

Vladimir Gurvich and Nhan Bao Ho; On tame, pet, domestic, and miserable impartial games; preprint in open access at RUTCOR Research Report, RRR-04-2015, Rutgers University.

Endre Boros, Vladimir Gurvich, Nhan Bao Ho, and Kazuhisa Makino; Extended complementary Nim; preprint in open access at RUTCOR Research Report, RRR-01-2015, Rutgers University.

Endre Boros, Vladimir Gurvich, Nhan Bao Ho, Kazuhisa Makino, and Peter Mursic; Two versions of the game of k-Nim; preprint in open access at RUTCOR Research Report, RRR-02-2015, Rutgers University.

Vladimir Gurvich; Further Generalizations of the Wythoff Game and Minimum Excludant; Discrete Applied Math. 160 (2012) 941–947; in open access at RUTCOR Research Report, RRR-16-2010 and revised at RRR-12-2011; Rutgers University.

Endre Boros, Vladimir Gurvich, and Vladimir Oudalov; A polynomial algorithm for a two-parameter extension of Wythoff NIM based on the Perron-Frobenius theory; Int. J. Game Theory 42:4 (2013) 891-915; in open access at RUTCOR Research Report, RRR-19-2011, Rutgers University.

Nhan Bao Ho; Two variants of Wythoff's game preserving its P-positions; J. Combin. Theory Ser. A 119 (2012) 1302–1314.

Israel M. Yaglom; Two games with matchsticks; in Serge Tabachnikov, Kvant Selecta: Combinatorics I, Volume 1, Mathematical world 17, American Mathematical Society (2001) 18; ISBN 9780821821718.