Vertex- and edge-transitivity in products of finite and infinite graphs

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Vertex transitivity of standard products of graphs is well investigated and understood. This topic comprises the first part of the talk. For finite graphs it is known that the Cartesian, the strong and the lexicographic product of two factors is vertex-transitive if and only if both factors are vertex transitive. For the direct product the result holds for products of connected, non-bipartite graphs. In the case of infinite graphs, on the other hand, the curious situation may occur that the so-called weak Cartesian product of infinitely many connected graphs is vertex-transitive, although all factors are asymmetric. An analogous result holds for the weak strong product of connected graphs [1]. Most of these results are collected in [4]. For more recent papers compare [3, 5, 8, 9].

Edge-transitivity of products, however, has only recently been dealt with, see for example [2, 6, 7]. Here the results differ widely between different types of products. We consider them in the second part of the talk.

We begin with edge-transitivity of products with finitely many factors, and then compare the results with those about products with infinitely many factors. Again it turns out that the differences can be significant. In particular, weak Cartesian products have a structure that differs markedly from that of products with finitely many factors. For example, every connected edge-transitive graph G that is not prime with respect to the Cartesian product is the Cartesian or weak Cartesian power of a connected, edge-transitive graph H. If G is a finite power of H, then H has to be vertex transitive. This is not the case if G is a weak Cartesian power of H. In this case H can have two vertex orbits, and G, although vertex- and edge-transitive, is only half-transitive.

Finally, several new results and open problems about products of directed and undirected graphs with finitely and infinitely many factors will be mentioned.

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