8th PhD Summer School in Discrete Mathematics
Questions on Colva's lectures on July 5th.

1. Show that a group of diagonal type is never 2-transitive.

2. Show that the intransitive subgroup $G = \mathrm{S}_k \times \mathrm{S}_{n-k}$ is maximal in S_n. [Hint: let $h \in \mathrm{S}_n \setminus G$. Show that all 2-cycles are in $\langle G, h \rangle$.]

3. Show that the imprimitive subgroup $G = \mathrm{S}_m \wr \mathrm{S}_2$ is maximal in S_{2m}. [Hint: use same approach as previous question].

4. Use the O'Nan–Scott Theorem to write down as many maximal subgroups of S_5 as you can. Can you prove your subgroups are maximal?

5. Show that $|\mathrm{PSL}_d(q)| = \frac{1}{(q-1,d)} q^{d(d-1)/2} \prod_{i=2}^{d} (q^i - 1)$.

6. Prove that $\mathrm{PSL}_2(3) \cong \mathrm{S}_4$, and that $\mathrm{PSL}_2(4) \cong \mathrm{A}_5$.

7. Prove that A_8 and $\mathrm{PSL}_3(4)$ have the same order, but are not isomorphic.