Recent results and open problems on unitals

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Graphs, groups, and more: celebrating Brian Alspach's 80th and Dragan Marušič's 65th birthdays

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It seems that there are many abstract unitals, but only a few embeddable in projective planes.

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Open question: Are there other unitals in $PG(2, q^2)$?

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Conjecture: This holds true for BM-unitals

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Open Problem: Condition on $Soc(G)$ is necessary?

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$\Rightarrow G \cong PSU(3,q), SU(3,q), Ree(q), PSL(2,q), SL(2,q)$, or G is sharply 2-transitive on ℓ.

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Conjecture If Γ is connected then $\mathcal{U}$ is isomorphic to the classical, or to a BM-unital.