Graphs, groups, and more: celebrating Brian Alspach's 80th and Dragan Marušič's 65th birthdays

Report of Contributions

Contribution ID: **0** Type: **not specified**

Classification of vertex-transitive digraphs via automorphism group

In the mid-1990s, two groups of authors independently obtained classifications of vertex-transitive graphs whose order is a product of two distinct primes. In the intervening years it has become clear that there is additional information concerning these graphs that would be useful, as well as making explicit the extensions of these results to digraphs. Additionally, there are several small errors in some of the papers that were involved in this classification. The purpose of this paper is to fill in the missing information as well as correct all known errors.

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Contribution ID: **1** Type: **not specified**

On the eigenvalues of Cayley graphs

In authors proposed a formula for computing the spectrum of Cayley graph $\Gamma = Cay(G, S)$ with respect to the character table of G where S is a symmetric normal subset of G.

Let q be a power of prime number p. A representation of degree n of group G is a homomorphism $\alpha : G \to GL(n, q)$, where $\alpha(g) = [g]_\beta$ for some basis β.
A character table is a matrix whose rows and columns are correspond to the irreducible characters and the conjugacy classes of G, respectively.

Let G be a group, for every element $g \in G$, we denote the conjugacy class of g by g^G. Assume that N be a normal subgroup of G and $\widetilde{\chi}$ is a character of G/N, then the character χ of G which is given by $\chi(g) = \widetilde{\chi}(Ng) \ \ \forall g \in Giscalledtheliftof \widetilde{\chi}$ to G.

Let G and H be two finite groups, then the direct product group $G \times H$ is a group whose elements are the Cartesian product of sets G, H and for $(g_1, h_1), (g_2, h_2) \in G \times H$ the related binary operation is defined as $(g_1, h_1)(g_2, h_2) = (g_1g_2, h_1h_2)$.

Theorem. Let G and H be two finite groups with irreducible characters $\varphi_1, \varphi_2, \cdots, \varphi_r$ and $\eta_1, \eta_2, \cdots, \eta_s$, respectively. Let $M(G)$ and $M(H)$ be character tables of G and H, respectively. Then the direct product group $G \times H$ has exactly rs irreducible characters $\varphi_i\eta_j$, where $1 \leq i \leq r$ and $1 \leq j \leq s$. In particular, the character table of group $G \times H$ is M(G×$H) = M(G) \otimes M(H), where \otimes$ denotes the Kronecker product.

[1] Diaconis, P., Shahshahani, M., (1981), Generating a random permutation with random transpositions, Zeit. fur Wahrscheinlichkeitstheorie verw. Gebiete, 57, pp. 159–179.

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Contribution ID: **2** Type: **not specified**

Characterization of generalized Petersen graphs that are Kronecker covers

The family of generalized Petersen graphs $G(n, k)$, introduced by Coxeter et al. [4] and named by Mark Watkins (1969), is a family of cubic graphs formed by connecting the vertices of a regular polygon to the corresponding vertices of a star polygon. The Kronecker cover $\mathrm{KC}(G)$ of a simple undirected graph G is a a special type of bipartite covering graph of G, isomorphic to the direct (tensor) product of G and K_2.
We characterize all the members of generalized Petersen graphs that are Kronecker covers, and describe the structure of their respective quotients.
We observe that some of such quotients are again generalized Petersen graphs, and describe all such pairs.

Primary authors: KRNC, Matjaž (University of Primorska); Prof. PISANSKI, Tomaž (University of Primorska)

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Contribution ID: **4** Type: **not specified**

New Properties of Realizations of Degree Sequences

Delen and Cangul recently defined a new graph invariant in terms of a degree sequence. By means of this invariant, it is possible to classify the realizations of the given degree sequence. Also this new invariant gives a lot of information about the connectedness, the number of components, loops, pendant edges, chords and multiple edges of the realizations. We shall give some new combinatorial and topological properties of this invariant.

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Presenter: Prof. CANGUL, Ismail Naci (Uludag University)

Contribution ID: **5** Type: **not specified**

Twofold triple systems that disprove Tutte's conjecture

The 2-*block intersection graph* (2-BIG) of a twofold triple system is the graph whose vertex set is the blocks of the TTS and two vertices are joined by an edge if they intersect in exactly two elements. A Hamilton cycle in a 2-BIG is equivalent to a cyclic Gray code, so an interesting problem is to classify which TTSs have Hamiltonian 2-BIGs.
The 2-BIGs are themselves interesting graphs: each component is cubic and 3-connected, and a 2-BIG is bipartite exactly when the TTS is decomposable to two Steiner triple systems. Any connected, bipartite 2-BIG with no Hamilton cycle is a counter-example to Tutte's conjecture.

Our main result is that for all $v \equiv 1 \text{ or } 3 \pmod 6$ such that $v > N$, there exists a simple, decomposable $TTS(v)$ whose 2-BIG is connected but not Hamiltonian.
N is currently about 700 but this has the potential to be improved.
Our result is achieved by embedding a simple, decomposable $TTS(u)$ with connected 2-BIG inside another simple, decomposable $TTS(v)$ with connected 2-BIG where $v > 2u + c$.
We also use a Tutte-like fragment to construct a decomposable, simple $TTS(331)$ whose 2-BIG is connected but not Hamiltonian.

Primary authors: PIKE, David (Memorial University); CAMERON, Rosalind (Memorial University)

Presenter: CAMERON, Rosalind (Memorial University)

Contribution ID: **6** Type: **not specified**

Counting Connected Sets and Connected Partitions of a Graph

Two related enumeration problems on vertex labeled graphs will be discussed. Given a graph G, we introduce and investigate the number $C(G)$ of connected subsets of the vertex set and the number $P(G)$ of connected partitions of the vertex set. By {\it connected} we mean that the induced subgraphs are connected. The numbers $C(G)$ and $P(G)$ can be regarded as the graph analogs of the number of subsets and the number of set partitions, respectively, of an n-element set.

Primary author: Prof. VINCE, Andrew (University of Florida)

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Contribution ID: **7** Type: **not specified**

On the isomorphisms of bi-Cayley graphs

Abstract

Let G be a group and S be a subset of G. Then $BCay(G,S)$, bi-Cayley graph of G with respect to S, is an undirected graph with vertex-set $G \times \{1,2\}$ and edge-set $\{\{(g,1),(sg,2)\} \mid g \in G, s \in S\}$. For $\sigma \in Aut(G)$ and $g \in G$, we have $BCay(G,S) \cong BCay(G,gS^\sigma)$. A bi-Cayley graph $BCay(G,S)$ is called a
BCI-graph if for any bi-Cayley graph $BCay(G,T)$, whenever $BCay(G,S) \cong BCay(G,T)$ we have $T = gS^\sigma$ for some $g \in G$ and $\sigma \in Aut(G)$. A group G is called a BCI-group if every bi-Cayley graph of G is a BCI-graph. In this lecture, we discuss recent results and future directions of classifying finite BCI-groups.

Introduction and results

A fundamental problem about Cayley graphs is the so called isomorphism problem, that is, given two Cayley graphs $Cay(G,S)$ and $Cay(H,T)$ determine whether or not $Cay(G,S) \cong Cay(H,T)$. It follows quickly from the definition that for any automorphism $\alpha \in Aut(G)$, the graphs $Cay(G,S)$ and $Cay(G,S^\alpha)$ are isomorphic,
namely, α induces an isomorphism between these graphs. Such an isomorphism is also called a *Cayley isomorphism*.

In 1967, Adam [1] conjectured that two Cayley graphs over the cyclic group are isomorphic if and only if there is a Cayley isomorphism which maps one to the other. Soon afterwards, Elspas and Turner [4] found the counterexample for $n = 8$. This also motivated the following definition. A Cayley graph $Cay(G,S)$ has the CI-property (for short, it is a CI-graph) if for any Cayley graph $Cay(G,T), Cay(G,S) \cong Cay(G,T)$ implies that $T = S^\alpha$ for some $\alpha \in Aut(G)$. Finite CI-groups have attracted considerable attention over the last 50 years. The problem of classifying finite CI-groups is still open.

In 2008, motivated by the concepts CI-graph, $m - BCI$-group and CI-group, Xu et al. [13] introduced the concepts BCI-graph, $m - BCI$-group and BCI-group, respectively. We say that a bi-Cayley graph $BCay(G,S)$ is a BCI-graph if whenever $BCay(G,S) \cong BCay(G,T)$ for some subset T of G, the set $T = gS^\alpha$ for some $g \in G$ and automorphism $\alpha \in Aut(G)$. The group G is an $m - BCI$-group if every bi-Cayley graph over G of valency at most m is a BCI-graph, and G is a BCI-group if every bi-Cayley graph over G is a BCI-graph. The theory of BCI-graphs and BCI-groups is less developed as in the case of CI-graphs and CI-groups. Several basic properties have been obtained by Jin and Liu in a series of papers [5-8], also by Koike et.al. [9-12] and very recently, by the present author [2-3]. We will discuss the relation between BCI-groups and CI-groups. In fact, our primary motivation by studying BCI-graphs and BCI-groups is that these objects can bring new insight into the old problem of classifying CI-groups. In [2] it is conjectured that every BCI-group is a CI-group and it is proved that every group of prime order is a BCI-group and every Sylow subgroup of a BCI-group is elementary abelian. Also, in[3], it is proved that every BCI-group is solvable.

Since every bi-Cayley graph over an abelian group is a Cayley graph over a generalized dihedral group, it seems that classifying finite abelian BCI-groups can help to classifying generalized dihedral CI-groups. In this lecture, we present some new results about classifying finite abelian BCI-groups.

This is a joint work with Majid Arezoomandb.

References:

[1] A. Adam. Research problems 2-10. J. Combin. Theory, 2, 393, 1967.

[2] M. Arezoomand and B. Taeri. Isomorphisms of finite semi-Cayley graphs. Acta Math. Sin. (Engl. Ser.), 31(4) 715-730, 2015.

[3] M. Arezoomand and B. Taeri. Finite BCI-groups are solvable. Int. J. Group Theory, 5(2), 1-6, 2016.

[4] B. Elspas and J. Turner. Graphs with circulants adjacency matrices. J. Combin. Theory, 9, 29-307, 1970.

[5]W. Jin and W. Liu. Two results on BCI-subset of finite groups. Ars Combin., 93,169-173, 2009.

[6]W. Jin and W. Liu, A classification of nonabelian simple 3-BCI-groups. European Journal of Combinatorics, 31 (2010) 1257-1264.

[7] W. Jin and W. Liu. On sylow subgroups of BCI-groups. Util. Math., 86,313-320, 2011.

[8] W. Jin and W. Liu. On isomorphisms of small order bi-Cayley graphs. Util. Math., 92, 317-327, 2013.

[9] H. Koike and I. Kovacs. Arc-transitive cubic abelian bi-cayley graphs and BCI-graphs. FILOMAT, 30(2) (2016) 321-331.

[10] H. Koike and I. Kovacs. A classification of nilpotent 3-BCI groups. submitted.

[11] H. Koike and I. Kovacs. Isomorphic tetravalent circulant Haar graphs. Ars Math. Contemp. 7(2), 215-235, 2014.

[12] H. Koike, I. Kovacs, and T. Pisanski. The number of cyclic configurations of type (v_3) and the isomorphism problem. J. Combin. Des., 22(5),216-229, 2014.

[13] S. J. Xu, W. Jin, Q. Shi, Y. Zhu and J. J. Li, The BCI-property of the Bi-Cayley graphs. J. Guangxi Norm. Univ.: Nat. Sci. Edition 26 (2008) 33-36.

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Presenter: Prof. IRANMANESH, Mohammad A. (Yazd University)

Contribution ID: **8** Type: **not specified**

Local actions in arc-transitive graphs

In this talk I'll give a partial (but possibly complete) answer to a question asked by Pierre-Emmanuel Caprace at the Groups St Andrews conference at Birmingham (UK) in August 2017, and investigated at the Tutte Centenary Retreat in Australia in November 2017.
Caprace asked if there exists a 2-transitive permutation group P such that only finitely many simple groups act arc-transitively on a symmetric graph X with local action P (of the stabiliser of a vertex v on the neighbourhood of˜v).
Some evidence is given to suggest that the answer is "No", even when "2-transitive" is replaced by "transitive". Indeed this will definitely be the answer if a highly likely conjecture about faithful quotients of amalgamated free products of groups is valid.
Then by way of illustration, I'll answer a follow-up question by showing that all but finitely many alternating groups have such an action on a 6-valent symmetric graph with vertex-stabiliser A_6.

Primary author: Prof. CONDER, Marston (University of Auckland)

Presenter: Prof. CONDER, Marston (University of Auckland)

Contribution ID: **9** Type: **not specified**

'Cayley graphs'? Who was Cayley?

Much of this meeting centres around Cayley graphs.
But who was Cayley? What contributions did he make to combinatorics? And what were his connections with graphs and groups?

Summary

This historical talk centres on Cayley and Cayley graphs.

Primary author: Prof. WILSON, Robin (The Open University, UK)

Presenter: Prof. WILSON, Robin (The Open University, UK)

Contribution ID: **10** Type: **not specified**

Automorphisms that preserve the natural edge-colouring

A major topic of research in symmetries of graphs is that of finding graphs that admit specific sorts of automorphisms, but do not admit other automorphisms. The study of regular representations is an example of this. Another way to approach this topic is to ask what constraints we can impose on a highly symmetric graph if we want to limit the automorphisms. It is with this approach in mind that we consider a natural edge-colouring.

Every Cayley graph comes with a natural edge-colouring that uses the elements of the connection set as the colours. It has long been known that for every connected Cayley digraph on a group G, the regular representation of G provides the only automorphisms that preserve this colouring. However, in the case of an undirected Cayley graph Cay(G, S) and $s \in S$, we must identify the colours s and s^{-1} since they apply to the same undirected edge. One effect of this is that some group automorphisms (those that map each $s \in S$ into $\{s, s^{-1}\}$) also preserve this colouring. Together with the regular representation of G, these are known as affine maps on the graph. In some cases there are additional graph automorphisms that are not affine maps.

We say that a Cayley graph is CCA, or has the CCA-property, if all of its colour-preserving automorphisms are affine. Similarly, we say that a group is CCA, or has the CCA-property, if every connected Cayley graph on that group is CCA. I will describe many of the known results on CCA groups and graphs, including joint work with Ted Dobson, Brandon Fuller, Ademir Hujdurovi\v{c}, Klavdija Kutnar, Luke Morgan, Dave Witte Morris, and Gabriel Verret.

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Presenter: MORRIS, Joy (University of Lethbridge, Canada)

Contribution ID: **11** Type: **not specified**

Some Variations on the Oberwolfach Theme

Last year marked the 50^{th} anniversary of the Oberwolfach Problem, which was formulated in 1967 by Gerhard Ringel as follows:

"At a conference in Oberwolfach, $2k+1$ participants are to be seated at t round tables for k meals so that each participant sits next to every other participant at exactly one meal. Can this be achieved with tables of sizes $m_1, m_2, \ldots, m_t$ if $m_1 + m_2 + \ldots + m_t = 2k+1$?"

This basic variant of the Oberwolfach Problem can be modeled as a decomposition of the complete graph K_{2k+1} into 2-factors, each of these 2-factors consisting of t disjoint cycles of lengths $m_1, m_2, \ldots, m_t$. The problem was later extended to $K_{2k+2} - I$, the complete graph of even order minus a 1-factor, and in this form nicknamed the Spouse-Avoiding Variant.

The case of uniform cycle lengths for both the original and the Spouse-Avoiding Variant was completely solved in a series of papers published between 1973 and 1991, and Brian Alspach co-authored two of the most comprehensive of these papers.
Over the last few decades, many other cases of the Oberwolfach Problem have been solved, however, the general problem is still open. In this talk, I will give an overview of the most prominent solved cases, and then focus on the variants of the problem that I have been involved with: the directed Oberwolfach Problem, the Spouse-Loving Variant (or the minimum covering variant), and the Honeymoon Oberwolfach Problem.

Primary author: Prof. SAJNA, Mateja (University of Ottawa)

Presenter: Prof. SAJNA, Mateja (University of Ottawa)

Contribution ID: **12** Type: **not specified**

Symmetry properties of generalized graph truncations

In the generalized graph truncation construction, one replaces each vertex of a k-regular graph Γ with a copy of a graph Υ of order k.
In this talk, which is based on joint work with Eduard Eiben and Robert Jajcay, we focus on symmetry properties of generalized truncations, especially in connection
to the symmetry properties of the graphs Γ and Υ used in the construction, and consider possible isomorphisms between different generalized truncations.
As an application, we obtain a classification of cubic vertex-transitive graphs of girths 3, 4, and 5.

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Presenter: Dr ŠPARL, Primož (University of Ljubljana and University of Primorska)

Contribution ID: **13** Type: **not specified**

Invariant generation of alternating groups by prime-power elements

Two elements x, y *invariantly generate* a group G if any conjugate x together with any conjugate of y generates G. Invariant generation of a group G by prime or prime-power elements has consequences for fixed-point-free actions on certain geometries with G actions.

In previous work, John Shareshian and I have shown that, assuming the Riemann hypothesis, the alternating groups A_n are invariantly generated by elements of prime order for all n except for n on a set of asymptotic density 0. On the other hand, we have constructed infinitely many examples that are not invariantly generated by such elements. I'll discuss ongoing work with Bob Guralnick and Shareshian, where we show that many alternating groups are invariantly generated by two elements of prime-power order.

Summary

Two elements x, y *invariantly generate* a group G if any conjugate x together with any conjugate of y generates G. I'll discuss invariant generation of alternating groups by two elements of prime-power order.

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Presenter: Dr WOODROOFE, Russ (University of Primorska)

Contribution ID: **14** Type: **not specified**

Enumerating locally restricted compositions over a finite group using de Bruijn graph and covering graph

Let $(\Gamma,+)$ be a finite group. An m-composition over Γ is an m-tuple $(g_1,g_2,\ldots,g_m)$ over Γ. It is called an m-composition of g if
$\sum_{j=1}^m g_j=g$. A composition (g_j) over Γ is called locally restricted if there is a positive integer σ such that any subsequence of (g_j) of length σ satisfies certain restrictions. Locally restricted compositions over Γ can be modeled using walks in a de Bruijin graph.
The de Bruijin graph over Γ with spanσ, denoted by $B(\Gamma;\sigma)$, is a digraph whose vertices are σ-tuples such that there is an arc from $\mathbf{u}:=(\mathbf{u}(1),\mathbf{u}(2),\ldots,\mathbf{u}(\sigma))$ to $\mathbf{v}:=(\mathbf{v}(1),\mathbf{v}(2),\ldots,\mathbf{v}(\sigma))$ if $\mathbf{v}(j)=\mathbf{u}(j+1)$, $1\le j\le\sigma-1$. Let D be a subgraph of $B(S;\sigma)$. We associate with each directed walk $\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_k$ in $B(\Gamma;\sigma)$ a composition
$\mathbf{c}=(\mathbf{v}_1(1),\ldots,\mathbf{v}_1(\sigma),\mathbf{v}_2(\sigma),\mathbf{v}_k(\sigma))$. That is, $\mathbf{c}$ is obtained from the walk by appending the last components of the subsequent vertices in the walk to the initial vertex of the walk.
We denote this set of compositions by $calC(D)$.

To keep track of the net sum of a composition in $calC(D)$, we make use of the derived graph of the voltage graph (D,α), where the voltage of the arc $(\mathbf{u},\mathbf{v})$
is given by $\alpha(\mathbf{u},\mathbf{v})=\mathbf{v}(\sigma)$. Let D' denote the derived graph of (D,α). That is, the vertex set of D' is $V(D)\times\Gamma$, and there is an arc
from $(\mathbf{u},g)$ to $(\mathbf{v},h)$ if and only if $(\mathbf{u},\mathbf{v})$ is an arc in D and $h=g+\mathbf{v}(\sigma)$. Let
$calS$ be the set of vertices in D' such that the second component is equal to
the sum of the parts of the first component.
It is easy to see that, for $m\ge\sigma$, an m-composition of g in $calC(D)$ corresponds to a walk
in D' from $calS$ to a vertex whose second component is g. Fix an ordering of the vertices of D' and let T denote the corresponding adjacency (transfer) matrix of D'. That is, $T(i,j)$ is equal to 1 if there is an arc from $\mathbf{v}_i$ to $\mathbf{v}_j$, and zero otherwise.

Let $\vec{s}$ denote the $\{0,1\}$ row vector such that its ith component is equal to 1 if and only if the corresponding vertex belongs to $calS$.
Let $\vec{f_g}$ denote the $\{0,1\}$ column vector such that its jth component is equal to 1 if and only if the corresponding vertex is of the form $(*,g)$. Then, for $m\ge\sigma$, the number of m-compositions of g in $calC(D)$ is equal to $\vec{s}M^{m-\sigma}\vec{f_g}$.

In this talk, we present some asymptotic results for the number of m-compositions, as $m\to\infty$, associated with some digraphs D and some finite group Γ. It will also be shown that the distribution of the number of occurrences of a given subword in a random locally restricted m-composition is asymptotically normal with mean and variance proportional to m. These results extend previous results on compositions over a finite abelian group. The basic tools for deriving these results are covering graphs of de Bruijn graphs, Perron-Frobenius theorem, and analytic combinatorics.

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Contribution ID: **15** Type: **not specified**

Self-orthogonal codes from orbit matrices of strongly regular graphs

In this talk we will show that under certain conditions submatrices of orbit matrices of strongly regular graphs span self-orthogonal codes. We apply this method to construct self-orthogonal binary linear codes from column orbit matrices of the triangular graph $T(2k)$ with at most 120 vertices.
Moreover, we construct linear codes from row orbit matrices of strongly regular graph with parameters (70,27,12,9). Further, we obtain strongly regular graphs and block designs from codewords of the constructed codes.
This is joint work with D. Crnković and S. Rukavina.

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Presenter: Dr MAKSIMOVIĆ, Marija (Department of Mathematics University of Rijeka)

Contribution ID: **16** Type: **not specified**

Integer flows in Cayley graphs

This is a survey talk about Tutte's integer flows in Cayley graphs. Alspach conjectured that every connected Cayley graph contains a Hamilton cycle. After almost five decades, Alspach's conjecture remains widely open. Note that every Hamiltonian graph admits a nowhere-zero 4-flow. The following is a weaker version of Alspach's conjecture (by Alspach, Liu and Z) that every Cayley graph admits a nowhere-zero 4-flow (equivalently, there is no Cayley snarks). Integer flow theory was introduced by Tutte as a dual version of graph coloring. Tutte proposed several conjectures about integer flows, such as, 3-, 4- and 5-flow conjecture. The progress of Tutte's conjectures for Cayley graphs will be surveyed and possible strengthening of those early results will be discussed based on
some recent progress in flow theory.

Primary author: Prof. ZHANG, Cun-Quan (West Virginia University)

Presenter: Prof. ZHANG, Cun-Quan (West Virginia University)

Contribution ID: **17** Type: **not specified**

Hamilton decompositions of one-ended Cayley graphs

In 1984, Alspach asked whether every Cayley graph of a finite Abelian group admits a Hamilton decomposition. The conjectured answer is yes, but except in some special cases the question remains wide open.

In this talk we study an analogous question for infinite, finitely generated groups, using spanning double rays as an infinite analogue of Hamilton cycles.

We show that if G is a one-ended Abelian group and S is a generating set only containing non-torsion elements, then the corresponding Cayley graph admits a decomposition into spanning double rays. In particular, any Cayley graph of $\mathbb{Z}^d$ has such a decomposition. Related results for two-ended groups will also be discussed.

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Presenter: LEHNER, Florian (University of Warwick)

Contribution ID: **19** Type: **not specified**

Self-dual codes from orbit matrices and quotient matrices of combinatorial designs

In this talk we will give constructions of self-orthogonal and self-dual codes, with respect to certain scalar products, with the help of orbit matrices of block designs and quotient matrices of symmetric (group) divisible designs (SGDDs) with the dual property. First we will describe constructions from block designs and their extended orbit matrices, where the orbit matrices are induced by the action of an automorphism group of the design. Then we will give some further constructions of self-dual codes specifically from symmetric block designs and their orbit matrices. Moreover, in a similar way as for symmetric designs, we will give constructions of self-dual codes from SGDDs with the dual property and their quotient matrices. This is joint work with Dean Crnković.

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Presenter: Dr MOSTARAC, Nina (Department of Mathematics, University of Rijeka)

Contribution ID: **20** Type: **not specified**

Graphs with small distinguishing index

\titleoftalk{Graphs with small distinguishing index}
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\begin{abstract}The distinguishing index of~a graph G, denoted by $D'(G)$, is the~least number of~colours in a general edge colouring of~G not preserved by any non-trivial automorphism. The definition of~$D'(G)$ was introduced in 2015 in [3] as an analogue of the distinguishing number defined by Albertson and Collins for vertex colouring, the concept of which spawned more than a hundred of papers.
For connected graphs in general, we showed in [3] that $D'(G) \leq \Delta(G)$ unless G is C_3, C_4 or C_5.
It was proved in [5],
that the equality $D'(G) = \Delta(G)$ holds only for cycles of~length at least 6, for K_4, $K_{3,3}$ and for all symmetric and bisymmetric trees, i.e., $D'(G) < \Delta(G)$ for all other connected graphs.
%\vspace{.3cm}

Interestingly, there are some wide classes of graphs with the distinguishing index bounded by a small constant, e.g., traceable graphs, planar graphs, claw-free graphs [5], Cartesian powers [2], and the Cartesian product of denumerable graphs [1].
\vspace{0.3cm}

An analogous concept was also investigated for proper total colourings in [4]. We
proved in particular that if G is a~connected graph such that its total chromatic index $\chi''(G)$ satisfies $\chi''(G) \geq \Delta(G) + 2$, then the total distinguishing chromatic index equals $\chi''(G)$.

\vspace{25pt}

\setlength{\parindent}{0cm}{\textbf{References:}
% Journal paper

[1] I. Broere, M. Pil\'sniak, {\it The distinguishing index of the Cartesian product of countable graphs}, Ars Math. Contemp. 13 (2017) 15–21.

[2] A. Gorzkowska, R. Kalinowski, M. Pil\'sniak, {\it The distinguishing index of the Cartesian product of graphs}, Ars Math. Contemp. 12 (2017) 77−-87.

[3] R.~Kalinowski and M.~Pil\'sniak, {\it Distinguishing graphs by edge colourings}, {European J. Combin.} {45} (2015) 124–131.

[4] R. Kalinowski, M. Pil\'sniak, M. Wo\'zniak, {\it Distinguishing graphs by total colourings}, {Ars Math. Contemp.}, (2016) 11:79−-89.

[5] M.~Pil\'sniak, {\it Improving Upper Bounds for the~Distinguishing Index}, {Ars Math. Contemp.} 13 (2017) 259–274.
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\end{abstract}

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Presenter: PILSNIAK, Monika (AGH University, Krakow, Poland)

Contribution ID: **21** Type: **not specified**

The Hamilton-Waterloo problem with cycle sizes of different parity

The Hamilton-Waterloo problem asks for a decomposition of the complete graph into r copies of a 2-factor F_1 and s copies of a 2-factor F_2 such that $r+s=\left\lfloor \frac{v-1}{2} \right\rfloor$. If F_1 consists of m-cycles and F_2 consists of n cycles, then we call such a decomposition a $(m,n)-\mathrm{HWP}(v;r,s)$. The goal is to find a decomposition for every possible pair (r,s). This problem has been studied in great depth in the cases when m and n have the same parity, but there are few general results for the case of different parity.
In this work, we use rings of polynomials of the form $\mathbb{Z}_{2^n}[x]/\left\langle x^2+x+1 \right\rangle$ to show that for odd x and y, there is a $(2^k x, y)-\mathrm{HWP}(vm;r,s)$ if $\gcd(x,y)\geq 3$, $m\geq 3$, and both x and y divide v, except possibly when $1\in\{r,s\}$.

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Co-author: KERANEN, Melissa (Michigan Technological University)

Presenter: Prof. PASTINE, Adrian (Universidad Nacional de San Luis)

Contribution ID: **22** Type: **not specified**

Bermond/Bollobas Problem and Ramanujan Graphs

If we denote by $n(k, d)$ the order of the largest undirected graphs of maximum degree k and diameter d, and let $M(k, d)$ denote the corresponding Moore bound, then $n(k, d) \leq M(k, d)$, for all $k \geq 3, d \geq 2$. While the inequality has been proven strict for all but very few pairs k and d, the exact relation between the values $n(k, d)$ and $M(k, d)$ is unknown, and the uncertainty of the situation is captured by a question of Bermond and Bollobas who asked whether it is true that for any a positive integer $c > 0$ there exist a pair k and d, such that $n(k, d) \leq M(k, d) - c$.

We show a surprising connection of this question to the value $2\sqrt{k-1}$, which is also essential in the definition of the Ramanujan graphs which are k-regular graphs having the property that their second largest eigenvalue (in modulus) does not exceed $2\sqrt{k-1}$. We further reinforce this surprising connection by showing an interesting consequence of a negative answer to the problem of Bermond and Bollobas. Namely, we show that if there exists a $c > 0$ such that $n(k, d) \geq M(k, d) - c$, for all $k \geq 3, d \geq 2$, then for any fixed k and all sufficiently large d's, the largest undirected graphs of degree k and diameter d must be Ramanujan graphs. Since Ramanujan graphs are scarce, our result seems to suggest a positive answer to the question of Bermond and Bollobas.

This is a joint work with Slobodan Filipovski.

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Co-author: Mr FILIPOVSKI, Slobodan (University of Primorska, Koper)

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Contribution ID: **23** Type: **not specified**

Some problems about symmetries of finite graphs

A few topics regarding symmetries of finite graphs that I find interesting, intriguing and worth studying shall be presented.

Summary

I would like to mention a few topics regarding symmetries of finite graphs that I find interesting, intriguing and worth studying.

The first topic is about lifting automorphisms along covering projections. Suppose one is given a finite connected graph Γ and a group of automorphisms G acting on it. Can one find a regular covering projection $\wp$ onto Γ such that G is the maximal group that lifts along $\wp$ and such that the full automorphism group of the graph is the lift of G? A recent partial result answer proved recently my Pablo Spiga and myself will be presented.

The second topic is about vertex-transitive graphs admitting an automorphism fixing many vertices; here a strict definition of the term "many" is intentionally avoided so that by varying it one can prove different results. Some computational data regarding cubic vertex-transitive graphs will be presented.

If time permits, a third topic regarding vertex-transitive graphs admitting an automorphism with a long orbit will be discussed; here the term "long" means a suitable fixed proportion of the order of the graph. Some results obtained recently by Micael Toledo about the cubic case will be mentioned.

Primary author: Mr POTOČNIK, Primož (University of Ljubljana)

Presenter: Mr POTOČNIK, Primož (University of Ljubljana)

Contribution ID: **24** Type: **not specified**

Recent results and open problems on unitals

A {\em unital} is defined to be a set of q^3+1 points equipped with a family of subsets, each of size $q+1$, such that every pair of distinct points are contained in exactly one subset of the family. Such subsets are usually called {\em blocks} so that unitals are block-designs $2-(q^3+1, q+1, 1)$. Unitals are known to play an important role in many investigations in finite geometry.
Computer aided searches suggest that there should be plenty of unitals, especially for small values of prime powers q, but those embeddable in a projective plane are quite rare.
Here a unital is {\em{embedded}} in a projective plane Π of order q^2 if its points and blocks are points and lines of Π.

Summary

We address the following topics

Recent results regarding the uniqueness of embedding of the classical and Buekenhout-Metz unitals in the Desarguesian plane of order q^2.

Automorphism groups of unitals containing translations, and a related open problem in Graph theory.

Primary author: Prof. KORCHMAROS, Gabor (University of Basilicata)

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Contribution ID: **25** Type: **not specified**

Triple intersection numbers of metric and cometric association schemes

An association scheme is called *metric*
if its intersection numbers satisfy the triangle inequality,
i.e., $p^h_{ij} \neq 0$ implies $|i - j| \leq h \leq i + j$
for some ordering of its relations.
Dually, an association scheme is called *cometric*
if its Krein parameters satisfy the triangle inequality
for some ordering of its eigenspaces.
Metric association schemes correspond precisely to distance-regular graphs,
and their parameters can be derived from a subset of the intersection numbers
which are usually written as the *intersection array*.
Similarly, the parameters of a cometric association scheme
can be computed from the *Krein array*.

A package for the Sage computer algebra system has been developed
for checking feasibility of a given intersection array
for a distance-regular graph.
It has been used to compute triple intersection numbers
for certain feasible intersection arrays,
from which nonexistence of the corresponding graphs has been then shown.

Recently, Williford [1] has published a list of feasible Krein arrays
for primitive 3-class cometric association scheme on up to 2800 vertices.
Gavrilyuk has suggested that the above mentioned software
be used to compute triple intersection numbers for the open cases in the list.
We have been able to use these computations to rule out several open cases.

This is joint work with Alexander Gavrilyuk.

[1] J. S. Williford. Primitive 3-class Q-polynomial association schemes, 2017.

Primary author: Dr VIDALI, Janoš (University of Ljubljana)

Presenter: Dr VIDALI, Janoš (University of Ljubljana)

Contribution ID: **26** Type: **not specified**

On decomposing 3-uniform hypergraphs into loose m-cycles

A *loose* m*-cycle* is a 3-uniform hypergraph with vertex set $\{v_1, v_2, \ldots, v_{2m}\}$ edge set $\{\{v_1, v_2, v_3\}, \{v_3, v_4, v_5\}, \ldots, \{v_{2m-1},$
We consider the problem of decomposing $K_v^{(3)}$, the complete 3-uniform hypergraph of order v, into edge-disjoint loose m-cycles. We settle the problem in the case $m = 4$.

Primary author: Prof. EL-ZANATI, Saad (Illinois State University)

Presenter: Prof. EL-ZANATI, Saad (Illinois State University)

Contribution ID: **27** Type: **not specified**

2-Limited Packings of Box Product Graphs

For a fixed integer k, a set of vertices B of a graph G is a k-*limited packing* of G provided that the closed neighourhood of any vertex in G contains at most k elements of B. The size of a largest possible k-limited packing in G is denoted $L_k(G)$ and is the k-limited packing number of G. In this talk, we investigate the 2-limited packing number of box products of paths. We show that the function \Delta[L_2(P_k \square P_n)] = L_2(P_k \square P_n) -L_2(P_k \square P_{n-1}) is eventually periodic, and thereby give closed formulas for L_2(P_k \square P_n), $k = 1, 2, \ldots, 5$. The techniques we use are suitable for establishing other types of packing and domination numbers for box products of paths and, more generally, for graphs of the form H \square P_n.

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Co-author: Dr GALLANT, Robert P. (Grenfell Campus, Memorial University)

Presenter: Dr CLARKE, Nancy E. (Acadia University)

Contribution ID: **28** Type: **not specified**

Decomposing complete multigraphs into stars of varying sizes

In 1979 Tarsi showed that an edge decomposition of a complete multigraph into stars of size k exists whenever the obvious necessary conditions hold. In 1996 Lin and Shyu gave necessary and sufficient conditions for the existence of an edge decomposition of a (simple) complete graph into stars of sizes $m_1, \ldots, m_t$.

I will discuss the common generalisation of these problems: when does a complete multigraph admit an edge decomposition into stars of sizes $m_1, \ldots, m_t$? This problem exhibits more complicated and interesting behaviour than either of its specialisations.

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Presenter: Dr HORSLEY, Daniel (Monash University)

Contribution ID: **29** Type: **not specified**

Majority coloring games

A vertex coloring of graph satisfies the *majority rule*, if for each vertex v at most half of its neighbors receive the same color as v. A coloring which satisfies the majority rule is called *majority coloring*. The problem of such colorings was introduced in [1,5] and continued with different variants in [2,4]. We consider its game version. For given graph G and color set C two players, Alice and Bob, in alternating turns color vertices with respect to the majority rule. Alice wins when every vertex becomes colored, while goal for Bob is to create a vertex which cannot be colored with any color belonging to the set C without breaking the majority rule. We show that if the color set C is finite, there exists a graph G on which Bob has winning strategy. Number of colors that Alice needs to have to win the game on graph G is clearly bounded by game coloring number of G. We improve that bound for some classes of graphs.

References:

[1] R. Aharoni, E.C. Milner and K. Prikry, Unfriendly partitions of a graph. Journal Combinatorial Theory (series B) 50 (1990), 1–10.

[2] M. Anholcer, B. Bosek and J. Grytczuk, Majority choosability of di-graphs. The Electronic Journal of Combinatorics 24 (3) (2017): #P3.57.

[3] T. Bartnicki, J. Grytczuk, H. A. Kierstead and X. Zhu, The map-coloring game. American Mathematical Monthly 114 (2007), 793–803.

[4] S. Kreutzer, S. Oum, P. Seymour, D. van der Zypen and D. R. Wood, Majority Closuring of Digraphs. arXiv:1608.03040 (2016).

[5] S. Shelah and E.C. Milner, Graphs with no unfriendly partitions. A tribute to Paul Erd\"os, Cambridge University Press, Cambridge (1990), 373–384.

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Presenter: JAKÓBCZAK, Gabriel (Jagiellonian University)

Contribution ID: **30** Type: **not specified**

Edge-critical graphs from the Möbius strip

Let χ'(G) denote the chromatic index of G and denote by G-e the graph obtained by removing an edge e from G. The graph G is said to be Δ-critical if χ'(G)=Δ+1 and
χ'(G-e)=Δ for any edge e in E(G). We are interested in critical graphs of even order.

It has been proved that there are no critical graphs of even order n≤ 10 and that there are no 3-critical graphs of order 12 and 14 (see [4]). Based on these results, Jakobsen formulated the famous critical graph conjecture, which claims that there are no critical graphs of even order (see [6]). A similar conjecture was made by Beineke and Wilson [1].

The graphs found by Goldberg [5], Fiol [3], Chetwynd and Wilson [2] disproved the conjecture. Goldberg constructed a 3-critical simple graph of order 22; Fiol found two
4-critical simple graphs of order 18 and 30; Chetwynd and Wilson found a 4-critical graph of order 16 with multiple edges.

Following the approach presented in [7], we construct infinite families of critical graphs of even order. Our method is based on a suitable identification of vertices which resembles the topological identification yielding the Möbius strip from a rectangular strip. By this method, we can also obtain the counterexamples found by Goldberg, Fiol, Chetwynd and Wilson.

REFERENCES

[1] L.W. Beineke, R.J. Wilson, On the edge-chromatic number of a graph, Discrete Math. 5 (1973), 15-20.

[2] A.G. Chetwynd, R.J. Wilson, The rise and fall of the critical graph conjecture, J. Graph Theory 7 (1983), 153-157.

[3] M.A. Fiol, 3-grafos criticos, Doctoral dissertation, Barcelona University, Spain (1980).

[4] S. Fiorini, R.J. Wilson, Edge-colourings of graphs, Research Notes in Mathematics, Vol. 16, Pitman, London (1977).

[5] M.K. Goldberg, On graphs of (maximum) degree 3 with chromatic index 4 (Russian), Bull. Acad. Sci. Georgia, SSR, 93 (1979), 29-31.

[6] I.T. Jakobsen, On critical graphs with chromatic index 4, Discrete Math. 9 (1974), 265-276.

[7] A. Vietri, An analogy between edge colourings and differentiable manifolds, with a new perspective on 3-critical graphs, Graphs Comb. 31 (2015), 2425-2435.

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Presenter: Dr BONVICINI, Simona (University of Modena and Reggio Emilia)

Contribution ID: **31** Type: **not specified**

Some bounds on the number of cyclic Steiner 2-designs

A $2-(v,k,1)$ design or, also, a Steiner 2-design is said to be cyclic if it admits an automorphism cyclically permuting all its points. To establish the number $\mathrm{NC}(v,k)$ of cyclic $2-(v,k,1)$ designs is in general not feasible and very little is known about this number. By "playing" with $(v,k,1)$ difference families, some lower bounds on $\mathrm{NC}(v,k)$ are given. In particular, for primes $p=6n+1$ with $p\equiv\pm1$ (mod 5), a construction involving the golden ratio of $\mathbb{Z}_p$ and the Narayana cows sequence is shown to give $\mathrm{NC}(p,3)>2^{3n/2}$.

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Presenter: Dr BRUGNOLI, Emanuele (University of Palermo)

Contribution ID: **32** Type: **not specified**

A Slavonian/Slovenian biographical history of configurations

This talk will discuss configurations and their history from a Slavonian, from a Slovenian, and from the author's point of view. Configurations are linear regular uniform hypergraphs, but investigated in a geometric language since 1876. Slavonia is a region in the east of Croatia, not to mix up with Slovenia, an independent state since 1991/1992. There are two recent books on configurations by the Slavonian Branko Gr\"unbaum, born in Osijek in 1929 "Configurations of points and lines", and by the Slovenian Toma\v z Pisanski, born in Ljubljana in 1949 "Configurations …..".

These configurations will be displayed in connection with the above mentioned books, with the author's own research and with the history of the last 30 years, seen through combinatorial conferences, e.g. 1988 in Ravello, 1991 in Bled, and further events in Slovenia.

This talk is motivated by publications of the Slavonian scholar Milutin Milankovi\'c (1879- 1958) such as
M.M.: Durch ferne Welten und Zeiten. Briefe einesWeltallbummlers, Leipzig (1936) (or the Serbian work of 1928), and M.M.: Zgodovina astronomije od njenih prvih začetkov do leta 1727, Ljubljana (1951).

In these two works M.M. mixes reports about astronomy, its history with biographical aspects of his own life.

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Presenter: GROPP, Harald (Universitaet Heidelberg)

Contribution ID: **33** Type: **not specified**

Hamilton Decompositions

I will talk about Hamilton decompositions of graphs, focussing on a number of results from the last few years. In particular I will discuss problems relating to Hamilton decompositions of vertex-transitive graphs and line graphs.

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Contribution ID: **35** Type: **not specified**

Graph-Theoretical Approaches in Drug Discovery at the PDB Scale

Proteins interact with other molecules through their protein binding sites, which are functionally important regions on the protein surface. Each binding site usually binds one or a few specific molecules, the ligands. Detection of binding sites gives insight in protein functionality and is hence essential for drug design. Sequence variants that occur in coding regions of genes may alter protein's amino acids and presumably affect protein function. It was found that disease-causing sequence variants are preferentially located at protein-protein interfaces rather than in noninterface regions of protein surfaces. Binding site sequence variants are of great interest to drug development.

Here we will describe newly developed algorithm and software, based on a novel three-dimensional graph representation of protein molecules and a fast maximum clique algorithm for binding site comparison in drug discovery at the PDB scale.

Summary

We have developed new methodological solutions for prediction and study of protein binding sites at the PDB scale, based on graph theoretical approaches, combined with molecular dynamics simulations. In particular, we have developed computational tools – **ProBiS tools** - which enable drug discovery based on protein structures: ProBiS algorithm, ProBiS, ProBiSCHARMMing, GenProBiS and ProBiS H2O web servers.

- ProBiS algorithm [1] detects and aligns similar binding sites based on graph theoretical approach - on our maximum clique algorithm [2], where proteins are represented as vertices and edges.
- ProBiS web server [3] detects structurally similar protein binding sites and predicts their ligands.
- ProBiSCHARMMing web server [4] predicts and minimizes ligands for any protein and can be used to prepare ligand-receptor complex for molecular dynamics simulation.
- GenProBiS web server [5] implements a novel approach to the discovery of sequence variants that have potentially deleterious effect on protein function and ligand binding through gain or loss of the binding site.
- ProBiS H2O web server [6] uses existing experimental structural data to identify conserved water sites on the interfaces of protein complexes, for example, protein-small molecule interfaces, and elsewhere on the protein structures.

Our newly developed approaches are particularly useful in the context of precision medicine. Our tools enable joining several otherwise disconnected areas of research, for example, graph-theoretical approaches, genome sequence studies, protein structures, and MD simulations [7].

References:

1. J. Konc, D. Janezic; *Bioinformatics*, **2010**, *26 (9),* 1160-1168.
2. J. Konc, D. Janezic; *MATCH*, **2007**, *58*, 569-590.
3. J. Konc, D. Janezic; *Nucleic Acids Research*, **2014**, *12*, W215–W220.
4. J. Konc, B.T. Miller, T. Stular, S. Lesnik, H.Lee Woodcock, B.R. Brooks, D. Janezic; *J. Chem. Info. Mod.,* **2015**, *55(11),* 2308-2314.

5. J. Konc, B. Skrlj, N. Erzen, T. Kunej, D. Janezic; *Nucleic Acids Research*, **2017**, *45*, W253-W259.
6. M. Jukic, J. Konc, S. Gobec, D. Janezic; *J. Chem. Info. Mod.*, **2017**, *57(12),* 3094-3103.
7. J. Konc, D. Janezic; *Progress in Biophysics and Molecular Biology*, **2017**, *128*, 24-32.

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Presenter: Prof. JANEŽIČ, Dušanka (UP FAMNIT)

Contribution ID: **36** Type: **not specified**

Quasi-semiregular automorphisms of cubic and tetravalent arc-transitive graphs

A non-trivial automorphism g of a graph Γ is called semiregular
if the only power g^i fixing a vertex is the identity mapping,
and it is called quasi-semiregular if it fixes one
vertex and the only power g^i fixing another vertex is the identity mapping. In this paper, we prove that K_4, the Petersen graph and the Coxeter graph are the only connected cubic arc-transitive graphs admitting a quasi-semiregular automorphism, and K_5 is the only connected tetravalent 2-arc-transitive graph admitting a quasi-semiregular automorphism. It will also be shown that every
connected tetravalent G-arc-transitive graph, where G is a solvable group containing a quasi-semiregular automorphism, is a normal Cayley graph of an abelian group of odd order.

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Presenter: KOVACS, Istvan

Contribution ID: **37** Type: **not specified**

On Orders of Vertex-stabilizers in Arc-transitive Graphs

One of the central questions in the study of graphs admitting a certain degree of symmetry is determining how large their automorphism groups can be. For graphs of fixed valency, this is equivalent with determining possible orders of vertex-stabilizers. The famous Tutte's result from 1948 implies that vertex stabilizer of a cubic arc-transitive graph can have order at most 48. Arc-transitive graphs of the same valency can have different local actions, hence the orders of vertex-stabilizers depend not only on valency but also on the local actions. Poto\v cnik, Spiga and Verret in 2014 characterized possible orders of vertex-stabilizers in arc-transitive graphs of valency at most 7 for all possible local actions, except for three 6-valent cases.
In this talk we prove that in these three remaining cases the order of vertex-stabilizers can be exponentially large (as a function of the number of vertices). For proving this we will use cubic-arc-transitive graphs with large eigenspaces.

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Presenter: Dr HUJDUROVIĆ, Ademir (University of Primorska)

Contribution ID: **38** Type: **not specified**

Patterns of Mirrors on Quasi-Platonic Surfaces

It is known that every Riemann surface of genus g can be expressed in the form $\mathbb{U}/\Omega$, where $\mathbb{U}$ is the Riemann sphere Σ, the Euclidean plane $\mathbb{C}$, or the hyperbolic plane $\mathbb{H}$, depending on whether g is 0, 1 or > 1, respectively, and Ω is a discrete group of isometries of $\mathbb{U}$.

A Riemann surface $S = \mathbb{U}/\Omega$ is \emph{quasi-Platonic} if Ω is normal in the ordinary triangle group $\Gamma[l, m, n]$. So S underlies a regular hypermap $\mathcal{H}$ of type (l, m, n). If Ω is also normal in the extended triangle group $\Gamma(l, m, n)$, then $\mathcal{H}$ is reflexible. Each reflection of $\mathcal{H}$ fixes a number of simple closed geodesics on S, which are called \emph{mirrors}. Then every mirror passes through some geometric points of $\mathcal{H}$ and these geometric points form a periodic sequence, which is called the \emph{pattern} of the mirror. By geometric points we mean the centers of the hypervertices, hyperedges and hyperfaces of $\mathcal{H}$.

In previous work David Singerman and I classified the patterns of mirrors on Platonic surfaces, which underlie regular maps, and in this work it is generalized to quasi-Platonic surfaces.

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Presenter: Dr MELEKOGLU, Adnan (Adnan Menderes University)

Contribution ID: **39** Type: **not specified**

Configurations of points and conics

A geometric configuration of type (p_q,n_k) is an incidence structure consisting of p points and n “blocks” such that each point is incident with q blocks and each block is incident with k points. The “blocks” can be different geometric figures such as lines, circles, conics, etc. The first known geometric configurations originate from classical incidence theorems such as the theorems of Pappus, Desargues, Miquel and Clifford.

In this talk we present examples of point-conic configurations, among them, some infinite classes, too. We discuss some relationships with other types of geometric configurations, as well as some open problems.

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Presenter: GÉVAY, Gábor (University of Szeged, Hungary)

Contribution ID: **40** Type: **not specified**

Forbidden Minors for Edge Searching

Edge searching is a combinatorial game where a team of searchers are following a strategy to capture a hidden intruder. It is played on the edges and vertices of a graph. When a fixed number of searchers are available, there is a finite number of forbidden minors, exclusion of which will guarantee that the graph will be intruder free.
In this talk, we give the set of forbidden minors for 4-searchable graphs of several graph classes: outerplanar graphs, series parallel graphs, and generalized wheel graphs. Finally, we will give a conjecture on the search number of circulant graphs of prime order.

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Co-authors: Prof. YANG, Boting (University of Regina); Dr DYER, Danny (Memorial University of Newfoundland)

Presenter: Dr DINER, Oznur Yasar (Kadir Has University)

Contribution ID: **41** Type: **not specified**

Orientable quadrilateral embeddings of cartesian products

White, Pisanski and others have proved a number of results on the existence of quadrilateral embeddings of cartesian products of graphs; in some cases these provide minimum genus embeddings. In a 1992 paper Pisanski posed three questions. First, if G and H are connected 1-factorable r-regular graphs with $r \geq 2$, does the cartesian product $G \times H$ have an orientable quadrilateral embedding? Second, if G is r-regular, does the cartesian product of G with sufficiently many even cycles have an orientable quadrilateral embedding? Third, if G is an arbitrary connected graph, does the cartesian product of G with a sufficient large cube $Q_n = \times^n K_2$ have an orientable quadrilateral embedding? We answer all three questions. The answers to the second and third questions are positive, as we show using a general theorem that answers both. We have also shown that the answer to the first question is negative, via some families of 3-regular examples.
This is joint work with Wenzhong Liu, Dong Ye and Xiaoya Zha.

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Presenter: Dr ELLINGHAM, Mark (Vanderbilt University, USA)

Contribution ID: **42** Type: **not specified**

Pursuit-evasion games and visibility

Broadly, there are two classical pursuit-evasion problems in graphs. One is cops and robbers, where cops move along vertices in pursuit of a robber that is slow, visible, and also moving on vertices. The other is edge-searching, or sweeping, where agents pursue a fast, invisible evader who moves on edges and vertices. This talk will talk about some of the history of these two problems, as they relate to Brian Alspach, and some recent results on zero-(and low-)visibility cops and robbers, which attempts to bridge the gaps between the two.

Primary author: DYER, Danny (Memorial University of Newfoundland)

Presenter: DYER, Danny (Memorial University of Newfoundland)

Contribution ID: **43** Type: **not specified**

The Distinguishing Index of 2-connected Graphs

The *distinguishing index* $D'(G)$ of a graph G is the least number of colours of an edge colouring that is not preserved by any non-trivial automorphism.

The following result was proved by Pil\'sniak in [1].

Theorem. If G is a connected graph, then
(1) $D'(G) = \Delta(G) + 1$ iff G is a cycle of length less than 6,
(2) $D'(G) = \Delta(G)$ iff G is a symmetric or a bisymmetric tree, a cycle of length at least 6, or K_4 or $K_{3,3}$,
(3) $D'(G) \leq \Delta(G) - 1$ otherwise.

In the same paper, Pil\'sniak formulated the following conjecture.

Conjecture. If a graph G is 2-connected, then D'(G)≤ $\left\lceil \sqrt{\Delta(G)} \right\rceil + 1$.

In this talk, we prove this conjecture in a bit stronger form, and show some of its consequences.

Reference:

[1] M. Pil\'sniak, Improving upper bounds for the distinguishing index, Ars Math. Contemp. 13 (2017) 259–274.

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Presenter: KALINOWSKI, Rafal (AGH University)

Contribution ID: **44** Type: **not specified**

Odd Cycle Bases of Nonbipartite Graphs

The cycle space of a graph G is the subspace of the edge space of G over the 2-element field that is spanned by the cycles of G (considered as edge-sets of G). Bondy and Lov\'{a}sz (1981) showed that the cycles through any set of $s-1$ vertices in an s-connected graph generate its cycle space. But what if these cycles are restricted to be of odd length?

In this talk, we consider this question and others regarding the cycle bases of nonbipartite graphs all of whose cycles are of odd length. It will be proved that if the graph is 3-connected and nonbipartite, then its cycle space can be generated by all odd cycles through a fix vertex. An example of Toft (1975) is used to show that two fixed vertices cannot be specified, contrasting the result of Bondy and Lov\'{a}sz when parity isn't a consideration. Other related results regarding the number of odd cycles in nonbipartite graphs will also be discussed.

Primary author: Prof. HARE, Donovan (University of British Columbia)

Presenter: Prof. HARE, Donovan (University of British Columbia)

Contribution ID: **45** Type: **not specified**

Upper bounds for the order of cages

A (k, g)-graph is a simple undirected k-regular graph with girth g. A (k, g)-cage is a (k, g)-graph with the least possible number of vertices. Cages have been studied quite intensively yet for most pairs (k, g) the corresponding cages are still unknown.

There is an obvious lower bound on the order of a (k, g)-cage, called the Moore bound (i.e., the number of vertices needed for a (k, g)-cage). There are several constructions for the upper bound for it, but often the upper bound is relatively far from the Moore bound.

In the talk I will discuss known upper bounds together with a new method for shortening the gap between the Moore bound and an upper bound on the order of a (k, g)-cage.

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Co-author: Prof. JAJCAY, Robert (Comenius University, Bratislava, and University of Primorska, Koper)

Presenter: Dr ZEMLJIC, Sara Sabrina (Comenius University Bratislava)

Contribution ID: **46** Type: **not specified**

On Reflexible Polynomials

A polynomial $f(x) = a_0 + a_1x + \ldots + a_nx^n$ over the prime field $\mathbb{Z}_p$, where p is odd, is reflexible if there exists $\lambda \in \mathbb{Z}_p^*$ such that $\lambda a_{n-i} = a_i$ (type 1) or $\lambda a_{n-i} = (-1)^i a_i$ (type 2), for all indices $i = 0, 1, \ldots, n$. Such polynomials were instrumental in the classification of quartic arc transitive graphs arising as minimal elementary abelian covers of doubled cycles, a problem that stems from an old result of Gardiner and Praeger.
Joint work with Bo\v stjan Kuzman and Primo\v z Poto\v cnik.

Primary author: MALNIC, Aleksander (University of Ljubljana & University of Primorska)

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Contribution ID: **47** Type: **not specified**

Estimation of the Recoverably from Discrete Presented Continuous Dependence

There is discussed the recovery conditions of continuous mapping from a discrete representation. The mapping is presented in a network of model elements located on a differentiable manifold.

Motivation for this material: a.) The applied models in geomechanics are characterized by large errors of the parameters and uncertainty in results. However, these models are presented by continuous dependencies. b.) The uncertainty principle is a fundamental concept in the context of signal and image processing. Uncertainty principles can be derived by using a group theoretic approach. This approach yields also the formalism in terms of continuous groups.

Homogeneous groups of symmetry: The method for valuation of dependencies is applied similar to "functional stability analysis" method, which is for to detect instability in numerical algorithms. This method has several key definitions. In the common theory of sensibility for computing problems are used valuations for precision. For optimal evaluation of the dependencies between physical values represented by differentiable manifold, the homogeneous groups of symmetry are used.

Application: The main links between the technical values existed because of the same basic physical values (M-mass, L-path and T-time) in their corresponding structural formulas. These relationships are expressed with the corresponding homogeneous groups of symmetry and invariants. Application is made for evaluation of the analytical expressions in a geomechanical model.

Primary author: Dr DIMITROV, Julian (Employer: University of Mining and Geology, Sofia, Bulgaria; Position: Chief assistant professor)

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Contribution ID: **48** Type: **not specified**

Steiner/Kirkman triple systems and their automorphism groups

I will survey known results and open problems about the automorphism groups of Steiner triple systems and Kirkman triple systems.

Primary author: Prof. BURATTI, Marco (Universita di Perugia)

Presenter: Prof. BURATTI, Marco (Universita di Perugia)

Contribution ID: **49** Type: **not specified**

Cayley Graphs Relative to Unions of Conjugacy Classes

The Cayley graph relative to a union of conjugacy classes of a finite group will be discussed: properties, results and open questions.

Primary author: Dr ZGRABLIC, Boris (University of Primorska)

Presenter: Dr ZGRABLIC, Boris (University of Primorska)

Contribution ID: **50** Type: **not specified**

Edge perturbations on signed graphs with clusters

Let Γ be a signed graph. A cluster in Γ of order c and degree s, is a pair of vertex subset (C, S), where C is a set of cardinality $c \geq 2$ of pairwise co-neighbor vertices sharing the same set of s neighbors and all edges connecting a fixed vertex in C are equallly signed. We consider the graph $\Gamma(H)$ which is obtained from G by identifying $V(H)$ with C and show that some Laplacian or Adjacency eigenvalues of $\Gamma(H)$ remain the same whatever H we choose in a suitable set of signed graphs.

Such techniques also provide a generalization to signed contexts of the Faria's lower bound on the multiplicity of the Laplacian eigenvalue 1 of a graph with pendant vertices.

Primary author: Dr BRUNETTI, Maurizio (Università di Napoli)

Co-author: BELARDO, Francesco (University of Naples Federico II)

Presenter: Dr BRUNETTI, Maurizio (Università di Napoli)

Contribution ID: **51** Type: **not specified**

Hamilton Paths and Cycles in Vertex-transitive Graphs

A path (cycle) containing every vertex in a graph is called a Hamilton path (Hamilton cycle, respectively). A graph is called vertex-transitive if for any pair of vertices u and v there exists an automorphism
mapping u to v. In 1969, Lovasz asked whether every finite connected vertex-transitive graph has a Hamilton path. With the exception of the complete graph on two vertices, only four connected vertex-transitive graphs that do not have a Hamilton cycle are known to exist. These four graphs are the Petersen graph, the Coxeter graph and the two graphs obtained from them by replacing each vertex by a triangle. The fact that none of these four graphs is a Cayley graph has led to a folklore conjecture that every Cayley graph has a Hamilton cycle. (A Cayley graph is a graph whose
automorphism group admits a regular subgroup.) Both of these two problems are still open. However, a considerable amount of partial results are known.

I will survey some results about the topic with special emphasis given to results obtained using methods initiated by Brian Alspach and/or Dragan Marušič.

Primary author: KUTNAR, Klavdija

Presenter: KUTNAR, Klavdija

Contribution ID: **52** Type: **not specified**

Hamilton cycles in cubic Cayley graphs of small girth

In 1969 Lov\' asz conjectured that a vertex transitive graph admits a hamilton path. In fact, only 5 non-hamiltonian vertex transitive graphs are known, namely K_2, the Petersen and the Coxeter graphs and their truncations. This motivates a folklore conjecture stating that every Cayley graph is hamiltonian. Moreover, four of the five examples are cubic graphs, therefore
it is natural to investigate the hamiltonicity of cubic Cayley graphs. In general, no non-hamiltonian cubic 7-cyclically connected graph except the Coxeter graph is known. This fact probably motivated C. Thomassen to state the following conjecture: If the cyclic connectivity of a cubic graph X is large enough, then X is hamiltonian. The conjecture can be strenghten as follows: Every 7-cyclically connected cubic graph except the Coxeter graph is hamiltonian. Since the cyclic connectivity of a vertex-transitive cubic graph is equal to the girth (Nedela, Skoviera 1995), there is a good motivation to investigate the hamiltonicity of vertex-transitive cubic graphs of girth at most six, an in particular, the cubic Cayley graphs of girth at most six. A small cycle in a Cayley graph implies a short relation in terms of generators, giving additional information on the base group. In the talk we use some results by Alspach and Maru\v si\v c to show that in most cases the cubic Cayley graphs of small girth are indeed hamiltonian, and we identify a few "hard families" of cubic Cayley graphs of small girth for which we are not able to verify the hamiltonicity.

The talk is based on a joint results done in collaboration with Elham Aboomahigir.

Primary author: NEDELA, Roman (University of West Bohemia)

Presenter: NEDELA, Roman (University of West Bohemia)