

Edge-critical graphs from the Möbius strip

Let $\chi'(G)$ denote the chromatic index of G and denote by $G-e$ the graph obtained by removing an edge e from G . The graph G is said to be Δ -critical if $\chi'(G)=\Delta+1$ and $\chi'(G-e)=\Delta$ for any edge e in $E(G)$. We are interested in critical graphs of even order.

It has been proved that there are no critical graphs of even order $n \leq 10$ and that there are no 3-critical graphs of order 12 and 14 (see [4]). Based on these results, Jakobsen formulated the famous critical graph conjecture, which claims that there are no critical graphs of even order (see [6]). A similar conjecture was made by Beineke and Wilson [1].

The graphs found by Goldberg [5], Fiol [3], Chetwynd and Wilson [2] disproved the conjecture. Goldberg constructed a 3-critical simple graph of order 22; Fiol found two 4-critical simple graphs of order 18 and 30; Chetwynd and Wilson found a 4-critical graph of order 16 with multiple edges.

Following the approach presented in [7], we construct infinite families of critical graphs of even order. Our method is based on a suitable identification of vertices which resembles the topological identification yielding the Möbius strip from a rectangular strip. By this method, we can also obtain the counterexamples found by Goldberg, Fiol, Chetwynd and Wilson.

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