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Enumerating locally restricted compositions over a finite group using de Bruijn graph and covering graph

Let $(\Gamma, +)$ be a finite group. An *m*-composition over Γ is an *m*-tuple (g_1, g_2, \ldots, g_m) over Γ . It is called an *m*-composition of *g* if

 $\sum_{i=1}^{m} g_i = g$. A composition (g_j) over Γ is called locally restricted if there is a positive integer σ

such that any subsequence of (g_j) of length σ satisfies certain restrictions. Locally restricted compositions over Γ can be modeled using walks in a de Bruijin graph.

The de Bruijin graph over Γ with span σ , denoted by $B(\Gamma; \sigma)$, is a digraph whose vertices are σ -tuples such that there is an arc from $\mathbf{u} := (\mathbf{u}(1), \mathbf{u}(2), \dots, \mathbf{u}(\sigma))$ to $\mathbf{v} := (\mathbf{v}(1), \mathbf{v}(2), \dots, \mathbf{v}(\sigma))$

if $\mathbf{v}(j) = \mathbf{u}(j+1)$, $1 \le j \le \sigma - 1$. Let D be a subgraph of $B(S; \sigma)$. We associate with each directed walk $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ in $B(\Gamma; \sigma)$ a composition

 $\mathbf{c} = (\mathbf{v}_1(1), \dots, \mathbf{v}_1(\sigma), \mathbf{v}_2(\sigma), \mathbf{v}_k(\sigma))$. That is, \mathbf{c} is obtained from the walk by appending the last components of the subsequent vertices in the walk to the initial vertex of the walk. We denote this set of compositions by calC(D).

The density and the compositions S_j case (S_j).

To keep track of the net sum of a composition in calC(D), we make use of the derived graph of the voltage graph (D, α) , where the voltage of the arc (\mathbf{u}, \mathbf{v})

is given by $\alpha(\mathbf{u}, \mathbf{v}) = \mathbf{v}(\sigma)$. Let D' denote the derived graph of (D, α) . That is, the vertex set of D' is $V(D) \times \Gamma$, and there is an arc

from (\mathbf{u}, g) to (\mathbf{v}, h) if and only if (\mathbf{u}, \mathbf{v}) is an arc in D and $h = g + \mathbf{v}(\sigma)$. Let

calS be the set of vertices in D' such that the second component is equal to

the sum of the parts of the first component.

It is easy to see that, for $m \ge \sigma$, an *m*-composition of *g* in calC(D) corresponds to a walk

in D' from calS to a vertex whose second component is g. Fix an ordering of the vertices of D' and let T denote the corresponding adjacency (transfer) matrix of D'. That is, T(i, j) is equal to 1 if there is an arc from \mathbf{v}_i to \mathbf{v}_j , and zero otherwise.

Let \vec{s} denote the $\{0, 1\}$ row vector such that its *i*th component is equal to 1 if and only if the corresponding vertex belongs to *calS*.

Let f_g denote the $\{0, 1\}$ column vector such that its *j*th component is equal to 1 if and only if the corresponding vertex is of the form (*, g). Then, for $m \ge \sigma$, the number of *m*-compositions of *g* in calC(D) is equal to $\vec{s}M^{m-\sigma}\vec{f_g}$.

In this talk, we present some asymptotic results for the number of m-compositions, as $m \to \infty$, associated with some digraphs D and some finite group Γ . It will also be shown that the distribution of the number of occurrences of a given subword in a random locally restricted m-composition is asymptotically normal with mean and variance proportional to m. These results extend previous results on compositions over a finite abelian group. The basic tools for deriving these results are covering graphs of de Bruijn graphs, Perron-Frobenius theorem, and analytic combinatorics.

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