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Enumerating locally restricted compositions over a finite group using de Bruijn graph and covering graph

Let $(\Gamma, +)$ be a finite group. An m-composition over Γ is an m-tuple $(g_1, g_2, \ldots, g_m)$ over Γ. It is called an m-composition of g if
$\sum_{j=1}^{m} g_j = g$. A composition (g_j) over Γ is called locally restricted if there is a positive integer σ
such that any subsequence of (g_j) of length σ satisfies certain restrictions. Locally restricted compositions over Γ can be modeled using walks in a de Bruijin graph.
The de Bruijin graph over Γ with spanσ, denoted by $B(\Gamma;\sigma)$, is a digraph whose vertices are σ-tuples such that there is an arc from $\mathbf{u} := (\mathbf{u}(1), \mathbf{u}(2), \ldots, \mathbf{u}(\sigma))$ to $\mathbf{v} := (\mathbf{v}(1), \mathbf{v}(2), \ldots, \mathbf{v}(\sigma))$
if $\mathbf{v}(j) = \mathbf{u}(j+1)$, $1 \le j \le \sigma - 1$. Let D be a subgraph of $B(S;\sigma)$. We associate with each directed walk $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ in $B(\Gamma;\sigma)$ a composition
$\mathbf{c} = (\mathbf{v}_1(1), \ldots, \mathbf{v}_1(\sigma), \mathbf{v}_2(\sigma), \mathbf{v}_k(\sigma))$. That is, $\mathbf{c}$ is obtained from the walk by appending the last components of the subsequent vertices in the walk to the initial vertex of the walk.
We denote this set of compositions by $calC(D)$.

To keep track of the net sum of a composition in $calC(D)$, we make use of the derived graph of the voltage graph (D, α), where the voltage of the arc $(\mathbf{u}, \mathbf{v})$
is given by $\alpha(\mathbf{u}, \mathbf{v}) = \mathbf{v}(\sigma)$. Let D' denote the derived graph of (D, α). That is, the vertex set of D' is $V(D) \times \Gamma$, and there is an arc
from $(\mathbf{u}, g)$ to $(\mathbf{v}, h)$ if and only if $(\mathbf{u}, \mathbf{v})$ is an arc in D and $h = g + \mathbf{v}(\sigma)$. Let
$calS$ be the set of vertices in D' such that the second component is equal to
the sum of the parts of the first component.
It is easy to see that, for $m \ge \sigma$, an m-composition of g in $calC(D)$ corresponds to a walk
in D' from $calS$ to a vertex whose second component is g. Fix an ordering of the vertices of D' and let T denote the corresponding adjacency (transfer) matrix of D'. That is, $T(i,j)$ is equal to 1 if there is an arc from $\mathbf{v}_i$ to $\mathbf{v}_j$, and zero otherwise.

Let $\vec{s}$ denote the $\{0,1\}$ row vector such that its ith component is equal to 1 if and only if the corresponding vertex belongs to $calS$.
Let $\vec{f_g}$ denote the $\{0,1\}$ column vector such that its jth component is equal to 1 if and only if the corresponding vertex is of the form $(*, g)$. Then, for $m \ge \sigma$, the number of m-compositions of g in $calC(D)$ is equal to $\vec{s} M^{m-\sigma} \vec{f_g}$.

In this talk, we present some asymptotic results for the number of m-compositions, as $m \to \infty$, associated with some digraphs D and some finite group Γ. It will also be shown that the distribution of the number of occurrences of a given subword in a random locally restricted m-composition is asymptotically normal with mean and variance proportional to m. These results extend previous results on compositions over a finite abelian group. The basic tools for deriving these results are covering graphs of de Bruijn graphs, Perron-Frobenius theorem, and analytic combinatorics.

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