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On the isomorphisms of bi-Cayley graphs

Abstract

Let G be a group and S be a subset of G. Then $BCay(G,S)$, bi-Cayley graph of G with respect to S, is an undirected graph with vertex-set $G\times\{1,2\}$ and edge-set $\{\{(g,1),(sg,2)\}\mid g\in G, s\in S\}$. For $\sigma\in Aut(G)$ and $g\in G$, we have $BCay(G,S)\cong BCay(G,gS^\sigma)$. A bi-Cayley graph $BCay(G,S)$ is called a BCI-graph if for any bi-Cayley graph $BCay(G,T)$, whenever $BCay(G,S)\cong BCay(G,T)$ we have $T=gS^\sigma$ for some $g\in G$ and $\sigma\in Aut(G)$. A group G is called a BCI-group if every bi-Cayley graph of G is a BCI-graph. In this lecture, we discuss recent results and future directions of classifying finite BCI-groups.

Introduction and results

A fundamental problem about Cayley graphs is the so called isomorphism problem, that is, given two Cayley graphs $Cay(G,S)$ and $Cay(H,T)$ determine whether or not $Cay(G,S)\cong Cay(H,T)$. It follows quickly from the definition that for any automorphism $\alpha\in Aut(G)$, the graphs $Cay(G,S)$ and $Cay(G,S^\alpha)$ are isomorphic,
namely, α induces an isomorphism between these graphs. Such an isomorphism is also called a *Cayley isomorphism.*

In 1967, Adam [1] conjectured that two Cayley graphs over the cyclic group are isomorphic if and only if there is a Cayley isomorphism which maps one to the other. Soon afterwards, Elspas and Turner [4] found the counterexample for $n=8$. This also motivated the following definition. A Cayley graph $Cay(G,S)$ has the CI-property (for short, it is a CI-graph) if for any Cayley graph $Cay(G,T), Cay(G,S)\cong Cay(G,T)$ implies that $T=S^\alpha$ for some $\alpha\in Aut(G)$. Finite CI-groups have attracted considerable attention over the last 50 years. The problem of classifying finite CI-groups is still open.

In 2008, motivated by the concepts CI-graph, $m-BCI$-group and CI-group, Xu et al. [13] introduced the concepts BCI-graph, $m-BCI$-group and BCI-group, respectively. We say that a bi-Cayley graph $BCay(G,S)$ is a BCI-graph if whenever $BCay(G,S)\cong BCay(G,T)$ for some subset T of G, the set $T=gS^\alpha$ for some $g\in G$ and automorphism $\alpha\in Aut(G)$. The group G is an $m-BCI$-group if every bi-Cayley graph over G of valency at most m is a BCI-graph, and G is a BCI-group if every bi-Cayley graph over G is a BCI-graph. The theory of BCI-graphs and BCI-groups is less developed as in the case of CI-graphs and CI-groups. Several basic properties have been obtained by Jin and Liu in a series of papers [5-8], also by Koike et.al. [9-12] and very recently, by the present author [2-3]. We will discuss the relation between BCI-groups and CI-groups. In fact, our primary motivation by studying BCI-graphs and BCI-groups is that these objects can bring new insight into the old problem of classifying CI-groups. In [2] it is conjectured that every BCI-group is a CI-group and it is proved that every group of prime order is a BCI-group and every Sylow subgroup of a BCI-group is elementary abelian. Also, in[3], it is proved that every BCI-group is solvable.

Since every bi-Cayley graph over an abelian group is a Cayley graph over a generalized dihedral group, it seems that classifying finite abelian BCI-groups can help to classifying generalized dihedral CI-groups. In this lecture, we present some new results about classifying finite abelian BCI-groups.

This is a joint work with Majid Arezoomandb.

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