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Twofold triple systems that disprove Tutte's conjecture

The 2-*block intersection graph* (2-BIG) of a twofold triple system is the graph whose vertex set is the blocks of the TTS and two vertices are joined by an edge if they intersect in exactly two elements.
A Hamilton cycle in a 2-BIG is equivalent to a cyclic Gray code, so an interesting problem is to classify which TTSs have Hamiltonian 2-BIGs.
The 2-BIGs are themselves interesting graphs: each component is cubic and 3-connected, and a 2-BIG is bipartite exactly when the TTS is decomposable to two Steiner triple systems. Any connected, bipartite 2-BIG with no Hamilton cycle is a counter-example to Tutte's conjecture.

Our main result is that for all $v \equiv 1 \text{ or } 3 \pmod{6}$ such that $v > N$, there exists a simple, decomposable $TTS(v)$ whose 2-BIG is connected but not Hamiltonian.
N is currently about 700 but this has the potential to be improved.
Our result is achieved by embedding a simple, decomposable $TTS(u)$ with connected 2-BIG inside another simple, decomposable $TTS(v)$ with connected 2-BIG where $v > 2u + c$.
We also use a Tutte-like fragment to construct a decomposable, simple $TTS(331)$ whose 2-BIG is connected but not Hamiltonian.

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