

On the eigenvalues of Cayley graphs

In authors proposed a formula for computing the spectrum of Cayley graph $\Gamma = \text{Cay}(G, S)$ with respect to the character table of G where S is a symmetric normal subset of G .

Let q be a power of prime number p . A representation of degree n of group G is a homomorphism $\alpha : G \rightarrow GL(n, q)$, where $\alpha(g) = [g]_\beta$ for some basis β .

A character table is a matrix whose rows and columns are correspond to the irreducible characters and the conjugacy classes of G , respectively.

Let G be a group, for every element $g \in G$, we denote the conjugacy class of g by g^G . Assume that N be a normal subgroup of G and $\tilde{\chi}$ is a character of G/N , then the character χ of G which is given by $\chi(g) = \tilde{\chi}(Ng) \quad \forall g \in G$ is called the lift of $\tilde{\chi}$ to G .

Let G and H be two finite groups, then the direct product group $G \times H$ is a group whose elements are the Cartesian product of sets G, H and for $(g_1, h_1), (g_2, h_2) \in G \times H$ the related binary operation is defined as $(g_1, h_1)(g_2, h_2) = (g_1g_2, h_1h_2)$.

Theorem. Let G and H be two finite groups with irreducible characters $\varphi_1, \varphi_2, \dots, \varphi_r$ and $\eta_1, \eta_2, \dots, \eta_s$, respectively. Let $M(G)$ and $M(H)$ be character tables of G and H , respectively. Then the direct product group $G \times H$ has exactly rs irreducible characters $\varphi_i\eta_j$, where $1 \leq i \leq r$ and $1 \leq j \leq s$. In particular, the character table of group $G \times H$ is $M(G \times H) = M(G) \otimes M(H)$, where \otimes denotes the Kronecker product.

[1] Diaconis, P., Shahshahani, M., (1981), Generating a random permutation with random transpositions, Zeit. für Wahrscheinlichkeitstheorie verw. Gebiete, 57, pp. 159–179.

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