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Bermond/Bollobas Problem and Ramanujan Graphs

If we denote by $n(k,d)$ the order of the largest undirected graphs of maximum degree k and diameter d, and let $M(k,d)$ denote the corresponding Moore bound, then $n(k,d) \le M(k,d)$, for all $k \ge 3, d \ge 2$. While the inequality has been proven strict for all but very few pairs k and d, the exact relation between the values $n(k,d)$ and $M(k,d)$ is unknown, and the uncertainty of the situation is captured by a question of Bermond and Bollobas who asked whether it is true that for any a positive integer $c > 0$ there exist a pair k and d, such that $n(k,d) \le M(k,d) - c$.

We show a surprising connection of this question to the value $2\sqrt{k-1}$, which is also essential in the definition of the Ramanujan graphs which are k-regular graphs having the property that their second largest eigenvalue (in modulus) does not exceed $2\sqrt{k-1}$. We further reinforce this surprising connection by showing an interesting consequence of a negative answer to the problem of Bermond and Bollobas. Namely, we show that if there exists a $c > 0$ such that $n(k,d) \ge M(k,d) - c$, for all $k \ge 3, d \ge 2$, then for any fixed k and all sufficiently large d's, the largest undirected graphs of degree k and diameter d must be Ramanujan graphs. Since Ramanujan graphs are scarce, our result seems to suggest a positive answer to the question of Bermond and Bollobas.

This is a joint work with Slobodan Filipovski.

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