Self-orthogonal codes from orbit matrices of strongly regular graphs

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SVEUČILIŠTE U RIJECI
ODJEL ZA MATEMATIKU

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Graphs, groups, and more: celebrating Brian Alspach's 80th and Dragan Marušič's 65th birthdays

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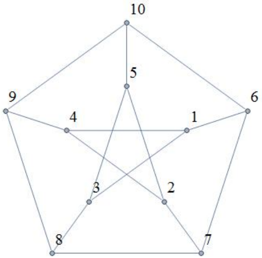
Definition

A simple regular graph is strongly regular with parameters (v, k, λ, μ) if it has v vertices, valency k, and if any two adjacent vertices are together adjacent to λ vertices, while any two non-adjacent vertices are together adjacent to μ vertices. A strongly regular graph with parameters (v, k, λ, μ) is usually denoted by $\mathrm{srg}(v, k, \lambda, \mu)$.

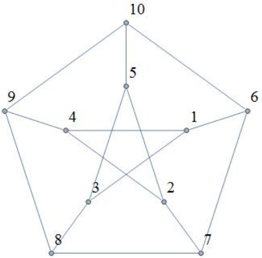
Definition

The adjacency matrix A of a graph Γ with v vertices is $v \times v$ matrix $M = (m_{ij})$ such that m_{ij} is number of edges incident with vertices x_i and x_j.

Petersen graph srg(10,3,0,1)



Petersen graph srg(10,3,0,1)



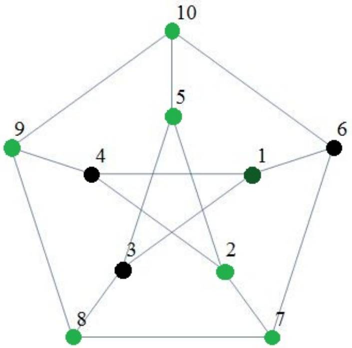
$$A = \begin{pmatrix}
0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0
\end{pmatrix}$$

Graph automorphism

An automorphism ρ of strongly regular graph Γ is a permutation on the set of vertices of a graph Γ such that for any two vertices of Γ u and v follows that: u and v are adjacent in Γ if and only if ρu and ρv are adjacent in Γ. Set of all automorphisms of strongly regular graph under the composition of functions forms a group that we call full automorphism group and denote $\mathrm{Aut}(\Gamma)$.

Example

Let an automorphism group G *generated with element* $\rho = (1)(3,4,6)(2,7,8,9,10,5)$ *partitions the set of vertices of Petersen graph into orbits* $O_1 = \{1\}, O_2 = \{3,4,6\}, O_3 = \{2,5,7,8,9,10\}$.



Example

	1	3	4	6	2	5	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
3	1	0	0	0	0	1	0	1	0	0
4	1	0	0	0	1	0	0	0	1	0
6	1	0	0	0	0	0	1	0	0	1
2	0	0	1	0	0	1	1	0	0	0
5	0	1	0	0	1	0	0	0	0	1
7	0	0	0	1	1	0	0	1	0	0
8	0	1	0	0	0	0	1	0	1	0
9	0	0	1	0	0	0	0	1	0	1
10	0	0	0	1	0	1	0	0	1	0

Example

	1	3	4	6	2	5	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
3	1	0	0	0	0	1	0	1	0	0
4	1	0	0	0	1	0	0	0	1	0
6	1	0	0	0	0	0	1	0	0	1
2	0	0	1	0	0	1	1	0	0	0
5	0	1	0	0	1	0	0	0	0	1
7	0	0	0	1	1	0	0	1	0	0
8	0	1	0	0	0	0	1	0	1	0
9	0	0	1	0	0	0	0	1	0	1
10	0	0	0	1	0	1	0	0	1	0

$$C = \begin{pmatrix} 0 & 1 & 0 \\ 3 & 0 & 1 \\ 0 & 2 & 2 \end{pmatrix}$$

Example

	1	3	4	6	2	5	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
3	1	0	0	0	0	1	0	1	0	0
4	1	0	0	0	1	0	0	0	1	0
6	1	0	0	0	0	0	1	0	0	1
2	0	0	1	0	0	1	1	0	0	0
5	0	1	0	0	1	0	0	0	0	1
7	0	0	0	1	1	0	0	1	0	0
8	0	1	0	0	0	0	1	0	1	0
9	0	0	1	0	0	0	0	1	0	1
10	0	0	0	1	0	1	0	0	1	0

$$R = \begin{pmatrix} 0 & 3 & 0 \\ 1 & 0 & 2 \\ 0 & 1 & 2 \end{pmatrix}$$

Definition

A $(b \times b)$-matrix $R = [r_{ij}]$ with entries satisfying conditions:

$$\sum_{j=1}^{b} r_{ij} = \sum_{i=1}^{b} \frac{n_i}{n_j} r_{ij} = k \tag{1}$$

$$\sum_{s=1}^{b} \frac{n_s}{n_j} r_{si} r_{sj} = \delta_{ij}(k - \mu) + \mu n_i + (\lambda - \mu) r_{ji} \tag{2}$$

where $0 \leq r_{ij} \leq n_j$, $0 \leq r_{ii} \leq n_i - 1$ and $\sum_{i=1}^{b} n_i = v$, is called a row orbit matrix for a strongly regular graph with parameters (v, k, λ, μ) and the orbit lengths distribution $(n_1, \ldots, n_b)$.

Definition

A $(b \times b)$-matrix $C = [c_{ij}]$ with entries satisfying conditions:

$$\sum_{i=1}^{b} c_{ij} = \sum_{j=1}^{b} \frac{n_j}{n_i} c_{ij} = k \tag{3}$$

$$\sum_{s=1}^{b} \frac{n_s}{n_j} c_{is} c_{js} = \delta_{ij}(k-\mu) + \mu n_i + (\lambda - \mu) c_{ij} \tag{4}$$

where $0 \leq c_{ij} \leq n_i$, $0 \leq c_{ii} \leq n_i - 1$ and $\sum_{i=1}^{b} n_i = v$, is called a column orbit matrix for a strongly regular graph with parameters (v, k, λ, μ) and the orbit lengths distribution $(n_1, \ldots, n_b)$.

Definition

A binary $[n, k]$ linear code C is a k-linear subspace of the vector space $\mathbb{F}_2^n$.

Definition

Let $x = (x_1, \dots x_n), \;\; y = (y_1, \dots y_n) \in \mathbb{F}_q^n$.

Hamming distance: $d(x, y) = |\{i \mid x_i \neq y_i, \;\; 1 \leq i \leq n\}|$.

Weight: $w(x) = d(x, 0) = |\{i \in \mathbb{N} \mid i \leq n, x_i \neq 0\}|$.

Minimum weight: $d = \min\{w(x) \mid x \in C, x \neq 0\}$

If a code C over a field of order q is of length n, dimension k, and minimum weight d, then we write $[n, k, d]_q$ to show this information.

Self-orthogonal codes

Definition
The dual code of a linear code $C \subset \mathbb{F}_q^n$ is the code $C^\perp \subset \mathbb{F}_q^n$ where

$$C^\perp = \{x \in F_q^n \mid x \cdot y = 0, \ \forall y \in C\}.$$

Definition
A code C is self-orthogonal if $C \subseteq C^\perp$.

Construction of self-orthogonal codes from fixed part of orbit matrices

Theorem

Let Γ be a $\mathrm{SRG}(v,k,\lambda,\mu)$ having an automorphism group G which acts on the set of vertices of Γ with b orbits of lengths $n_1,\ldots,n_b$, respectively, with f fixed vertices, and the other $b-f$ orbits of lengths $n_{f+1},\ldots,n_b$ divisible by p, where p is a prime dividing k, λ and μ. Let C be the column orbit matrix of the graph Γ with respect to G. If q is a prime power such that $q=p^n$, then the code spanned by the rows of the fixed part of the matrix C is a self-orthogonal code of length f over F_q.

	1	$\cdots$	1	n_{f+1}	$\ldots$	n_b
1						
$\vdots$						
1						
n_{f+1}						
$\vdots$						
n_b						

Results

Table: Codes from the fixed parts of orbit matrices for Z_2 acting on $T(2k)$, $3 \leq k \leq 8$

$T(2k)$	C	$\lvert\mathrm{Aut}(C)\rvert$	Weight Distribution
$3 \leq k \leq 8$	$[k+4, 2, 4]$	$2\cdot\ 4!(k\text{-}2)!$	$[<0,1>,<4,3>]$
$4 \leq k \leq 8$	$[k+12, 4, 8]$	$4\cdot 7!(k\text{-}3)!$	$[<0,1>,<8,15>]$
$5 \leq k \leq 8$	$[k+24, 6, 12]$	$8!(k\text{-}4)!$	$[<0,1>,<12,28>,<16,35>]$
$6 \leq k \leq 8$	$[k+40, 8, 16]$	$10!(k\text{-}5)!$	$[<0,1>,<16,45>,<24,210>]$
$7 \leq k \leq 8$	$[k+60, 10, 20]$	$12!(k\text{-}6)!$	$[<0,1>,<20,66>,<32,495>,<36,462>]$
$k = 8$	$[k+84, 12, 24]$	$14!(k\text{-}7)!$	$[<0,1>,<24,91>,<40,1001>,<48,3003>]$

Results

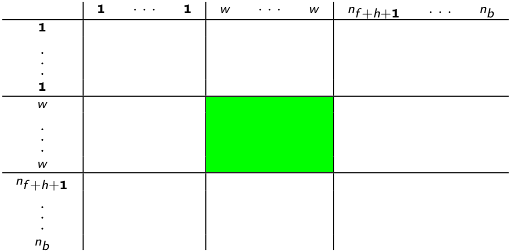
Table: Codes from the fixed part of orbit matrices for Z_4 acting on $T(2k)$, $3 \leq k \leq 8$

$T(2k)$	C	$\|Aut(C)\|$	Weight Distribution
$k = 4, 6, 8$	[6,2,4]	$2^4 3^1$	$[< 0, 1 >, < 4, 3 >]$
$k = 5, 7$	[7,2,4]	$2^4 3^1$	$[< 0, 1 >, < 4, 3 >]$
$k = 6, 8$	[8,2,4]	$2^5 3^1$	$[< 0, 1 >, < 4, 3 >]$
$k = 7, 8$	[k+2,2,4]	$2^{2k-9} 3^2$	$[< 0, 1 >, < 4, 3 >]$
$k = 5, 7$	[15,4,8]	$2^6 3^2 5^1 7^1$	$[< 0, 1 >, < 8, 15 >]$
$k = 6, 8$	[16,4,8]	$2^6 3^2 5^1 7^1$	$[< 0, 1 >, < 8, 15 >]$
$k = 7, 8$	[k+10,4,8]	$2^7 3^{k-5} 5^1 7^1$	$[< 0, 1 >, < 8, 15 >]$
$k = 6, 8$	[28,6,12]	$2^7 3^2 5^1 7^1$	$[< 0, 1 >, < 12, 28 >, < 16, 35 >]$
$k = 7, 8$	[k+22,6,12]	$2^k 3^2 5^1 7^1$	$[< 0, 1 >, < 12, 28 >, < 16, 35 >]$
$k = 7, 8$	[k+38,8,16]	$2^8 3^4 5^2 7^1$	$[< 0, 1 >, < 16, 45 >, < 24, 210 >]$
$k = 8$	[66,10,20]	$2^{10} 3^5 5^2 7^1 11^1$	$[< 0, 1 >, < 20, 66 >, < 32, 495 >, < 36, 462 >]$

Construction of self-orthogonal codes from nonfixed part of orbit matrices

Theorem

Let Γ be a $\mathrm{SRG}(v,k,\lambda,\mu)$ having an automorphism group G which acts on the set of vertices of Γ with b orbits of lengths $n_1,\dots,n_b$, respectively, such that there are f fixed vertices, h orbits of length w, and $b-f-h$ orbits of lengths $n_{f+h+1},\dots,n_b$. Further, let $pw|n_s$ if $w<n_s$, and $pn_s|w$ if $n_s<w$, for $s=f+h+1,\dots,b$, where p is a prime number dividing k, λ, μ and w. Let C be the column orbit matrix of the graph Γ with respect to G. If q is a prime power such that $q=p^n$, then the code over F_q spanned by the part of the matrix C (rows and columns) determined by the orbits of length w is a self-orthogonal code of length h.



Results

Table: Codes from the nonfixed parts of orbit matrices for Z_2 acting on $T(2k)$, $3 \leq k \leq 8$

$T(n)$	C	$\lvert\mathrm{Aut}(C)\rvert$	WeightDistribution
$T(6)$	[6,2,4]	$2^{4}3$	$[< 0,1 >, < 4,3 >]$
$T(8)$	[10,2,6]	$2^{8}3^{2}$	$[< 0,1 >, < 6,2 >, < 8,1 >]$
$T(8)$	[12,2,8]	$2^{10}3^{4}$	$[< 0,1 >, < 8,3 >]$
$T(8)$	[12,3,6]	$2^{9}3$	$[< 0,1 >, < 6,4 >, < 8,3 >]$
$T(10)$	[14,2,8]	$2^{10}3^{4}5^{2}$	$[< 0,1 >, < 8,2 >, < 12,1 >]$
$T(10)$	[18,3,8]	$2^{13}3^{4}$	$[< 0,1 >, < 8,3 >, < 12,4 >]$
$T(10)$	[20,4,8]	$2^{13}3^{1}5^{1}$	$[< 0,1 >, < 8,5 >, < 12,10 >]$
$T(12)$	[18,2,10]	$2^{16}3^{4}5^{2}7^{2}$	$[< 0,1 >, < 10,2 >, < 16,1 >]$
$T(12)$	[24,3,10]	$2^{16}3^{7}5^{3}$	$[< 0,1 >, < 10,3 >, < 16,3 >, < 18,1 >]$
$T(12)$	[28,4,10]	$2^{21}3^{5}$	$[< 0,1 >, < 10,4 >, < 16,7 >, < 18,4 >]$
$T(12)$	[30,5,10]	$2^{19}3^{2}5^{1}$	$[< 0,1 >, < 10,6 >, < 16,15 >, < 18,10 >]$
$T(12)$	[30,4,16]	$2^{21}3^{2}5^{1}7^{1}$	$[< 0,1 >, < 16,15 >]$
$T(14)$	[22,2,12]	$2^{18}3^{8}5^{4}7^{2}$	$[< 0,1 >, < 12,2 >, < 20,1 >]$
$T(14)$	[30,3,12]	$2^{25}3^{7}5^{3}7^{3}$	$[< 0,1 >, < 12,3 >, < 20,3 >, < 24,1 >]$
$T(14)$	[36,4,12]	$2^{25}3^{9}5^{4}$	$[< 0,1 >, < 12,4 >, < 20,6 >, < 24,5 >]$
$T(14)$	[40,5,12]	$2^{28}3^{6}5^{1}$	$[< 0,1 >, < 12,5 >, < 20,11 >, < 24,15 >]$
$T(14)$	[42,6,12]	$2^{25}3^{2}5^{1}7^{1}$	$[< 0,1 >, < 12,7 >, < 20,21 >, < 24,35 >]$
$T(16)$	[26,2,14]	$2^{22}3^{10}5^{4}7^{2}11^{2}$	$[< 0,1 >, < 14,2 >, < 24,1 >]$
$T(16)$	[36,3,14]	$2^{28}3^{13}5^{6}7^{3}$	$[< 0,1 >, < 14,3 >, < 24,3 >, < 30,1 >]$
$T(16)$	[44,4,14]	$2^{37}3^{9}5^{4}7^{4}$	$[< 0,1 >, < 14,4 >, < 24,6 >, < 30,4 >, < 32,1 >]$
$T(16)$	[50,5,14]	$2^{33}3^{11}5^{6}$	$[< 0,1 >, < 14,5 >, < 24,10 >, < 30,11 >, < 32,5 >]$
$T(16)$	[54,6,14]	$2^{37}3^{8}5^{1}$	$[< 0,1 >, < 14,6 >, < 24,16 >, < 30,26 >, < 32,15 >]$
$T(16)$	[56,6,24]	$2^{35}3^{2}5^{1}7^{1}$	$[< 0,1 >, < 24,28 >, < 32,35 >]$
$T(16)$	[56,7,14]	$2^{35}3^{2}5^{1}7^{1}$	$[< 0,1 >, < 14,8 >, < 24,28 >, < 30,56 >, < 32,35 >]$

Results

Table: Codes from parts of orbit matrices for Z_4 corresponding to the orbits of length 2

$T(n)$	C	$\|\mathrm{Aut}(C)\|$	Weight Distribution
$T(\mathbf{10})$	[7,2,4]	$\mathbf{2^4 3^1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{4},\mathbf{3}>]$
$T(\mathbf{12})$	[11,2,6]	$\mathbf{2^8 3^2}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{6},\mathbf{2}>,<\mathbf{8},\mathbf{1}>]$
$T(\mathbf{12})$	[13,2,8]	$\mathbf{2^{10} 3^4}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{8},\mathbf{3}>]$
$T(\mathbf{12})$	[13,3,6]	$\mathbf{2^9 3^1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{6},\mathbf{4}>,<\mathbf{8},\mathbf{3}>]$
$T(\mathbf{14})$	[8,2,4]	$\mathbf{2^5 3^1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{4},\mathbf{3}>]$
$T(\mathbf{14})$	[15,2,8]	$\mathbf{2^{10} 3^4 5^2}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{8},\mathbf{2}>,<\mathbf{12},\mathbf{1}>]$
$T(\mathbf{14})$	[19,3,8]	$\mathbf{2^{13} 3^4}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{8},\mathbf{3}>,<\mathbf{12},\mathbf{4}>]$
$T(\mathbf{14})$	[21,4,8]	$\mathbf{2^{13} 3^1 5^1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{8},\mathbf{5}>,<\mathbf{12},\mathbf{10}>]$
$T(\mathbf{16})$	[12,2,6]	$\mathbf{2^9 3^2}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{6},\mathbf{2}>,<\mathbf{8},\mathbf{1}>]$
$T(\mathbf{16})$	[14,2,8]	$\mathbf{2^{11} 3^4}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{8},\mathbf{3}>]$
$T(\mathbf{16})$	[14,3,6]	$\mathbf{2^{10} 3^1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{6},\mathbf{4}>,<\mathbf{8},\mathbf{3}>]$
$T(\mathbf{16})$	[19,2,10]	$\mathbf{2^{16} 3^4 5^2 7^2}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{10},\mathbf{2}>,<\mathbf{16},\mathbf{1}>]$
$T(\mathbf{16})$	[25,3,10]	$\mathbf{2^{16} 3^7 5^3}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{10},\mathbf{3}>,<\mathbf{16},\mathbf{3}>,<\mathbf{18},\mathbf{1}>]$
$T(\mathbf{16})$	[29,4,10]	$\mathbf{2^{21} 3^5}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{10},\mathbf{4}>,<\mathbf{16},\mathbf{7}>,<\mathbf{18},\mathbf{4}>]$
$T(\mathbf{16})$	[31,4,16]	$\mathbf{2^{21} 3^2 5^1 7^1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{16},\mathbf{15}>]$
$T(\mathbf{16})$	[31,5,10]	$\mathbf{2^{19} 3^2 5^1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{10},\mathbf{6}>,<\mathbf{16},\mathbf{15}>,<\mathbf{18},\mathbf{10}>]$

Results

Table: Codes from parts of orbit matrices for Z_4 corresponding to the orbits of length 4

$T(n)$	C	$\|\mathrm{Aut}(C)\|$	Weight Distribution
$T(\mathbf{10})$	[10,2,4]	$\mathbf{2^7 3^1}$	$[<\mathbf{0},\mathbf{1}>, <\mathbf{4},\mathbf{1}>, <\mathbf{6},\mathbf{2}>]$
$T(\mathbf{12})$	[14,2,8]	$\mathbf{2^{11} 3^4}$	$[<\mathbf{0},\mathbf{1}>, <\mathbf{8},\mathbf{3}>]$
$T(\mathbf{12})$	[15,2,8]	$\mathbf{2^{11} 3^5}$	$[<\mathbf{0},\mathbf{1}>, <\mathbf{8},\mathbf{3}>]$
$T(\mathbf{14})$	[18,2,10]	$\mathbf{2^{13} 3^5 5^2}$	$[<\mathbf{0},\mathbf{1}>, <\mathbf{10},\mathbf{2}>, <\mathbf{12},\mathbf{1}>]$
$T(\mathbf{14})$	[21,3,6]	$\mathbf{2^{14} 3^5}$	$[<\mathbf{0},\mathbf{1}>, <\mathbf{6},\mathbf{1}>, <\mathbf{10},\mathbf{3}>, <\mathbf{12},\mathbf{3}>]$
$T(\mathbf{16})$	[22,2,12]	$\mathbf{2^{19} 3^5 5^2 7^2}$	$[<\mathbf{0},\mathbf{1}>, <\mathbf{12},\mathbf{2}>, <\mathbf{16},\mathbf{1}>]$
$T(\mathbf{16})$	[27,3,12]	$\mathbf{2^{22} 3^8}$	$[<\mathbf{0},\mathbf{1}>, <\mathbf{12},\mathbf{4}>, <\mathbf{16},\mathbf{3}>]$
$T(\mathbf{16})$	[28,2,16]	$\mathbf{2^{25} 3^8 5^3 7^3}$	$[<\mathbf{0},\mathbf{1}>, <\mathbf{16},\mathbf{3}>]$

Theorem

Let Γ be a $\mathrm{SRG}(v,k,\lambda,\mu)$ with an automorphism group G which acts on the set of vertices of Γ with b orbits of lengths $n_1,\ldots,n_b$, respectively, and $w = max\{n_1,\ldots,n_b\}$. Further, let p be a prime dividing k, λ, μ and w, and let $pn_s|w$ if $n_s \neq w$. Let C be the column orbit matrix of the graph Γ with respect to G. If q is a prime power such that $q = p^n$, then the code over F_q spanned by the rows of C corresponding to the orbits of length w is a self-orthogonal code of length b.

	n_1	$\cdots$	n_b	w	$\cdots$	w
n_1						
$\vdots$						
n_b						
w						
$\vdots$						
w						

Results

Table: Codes from orbit matrices for Z_4 spanned by the rows corresponding to the orbits of length 4

$T(n)$	C	$\lvert \mathrm{Aut}(C) \rvert$	WeightDistribution
$T(\mathbf{10})$	[13,2,6]	$2^{9}3^{2}5^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{4},\mathbf{1}>,<\mathbf{6},\mathbf{2}>]$
$T(\mathbf{12})$	[18,2,8]	$2^{14}3^{6}5^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{8},\mathbf{3}>]$
$T(\mathbf{12})$	[20,2,8]	$2^{17}3^{6}5^{1}7^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{8},\mathbf{3}>]$
$T(\mathbf{12})$	[22,2,8]	$2^{18}3^{8}5^{2}7^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{8},\mathbf{3}>]$
$T(\mathbf{14})$	[25,3,6]	$2^{17}3^{6}5^{1}7^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{6},\mathbf{1}>,<\mathbf{10},\mathbf{3}>,<\mathbf{12},\mathbf{3}>]$
$T(\mathbf{14})$	[29,2,10]	$2^{22}3^{10}5^{4}7^{1}11^{1}13^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{10},\mathbf{2}>,<\mathbf{12},\mathbf{1}>]$
$T(\mathbf{14})$	[31,2,10]	$2^{23}3^{11}5^{5}7^{2}11^{1}13^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{10},\mathbf{2}>,<\mathbf{12},\mathbf{1}>]$
$T(\mathbf{14})$	[35,2,10]	$2^{28}3^{13}5^{5}7^{2}11^{1}13^{1}17^{1}19^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{10},\mathbf{2}>,<\mathbf{12},\mathbf{1}>]$
$T(\mathbf{16})$	[32,2,16]	$2^{29}3^{9}5^{4}7^{4}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{16},\mathbf{3}>]$
$T(\mathbf{16})$	[34,3,12]	$2^{29}3^{11}5^{2}7^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{12},\mathbf{4}>,<\mathbf{16},\mathbf{3}>]$
$T(\mathbf{16})$	[36,3,12]	$2^{31}3^{12}5^{2}7^{1}11^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{12},\mathbf{4}>,<\mathbf{16},\mathbf{3}>]$
$T(\mathbf{16})$	[40,2,12]	$2^{36}3^{13}5^{6}7^{4}11^{1}13^{1}17^{1}19^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{12},\mathbf{2}>,<\mathbf{16},\mathbf{1}>]$
$T(\mathbf{16})$	[42,2,12]	$2^{37}3^{14}5^{6}7^{5}11^{2}13^{1}17^{1}19^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{12},\mathbf{2}>,<\mathbf{16},\mathbf{1}>]$
$T(\mathbf{16})$	[46,2,12]	$2^{41}3^{15}5^{8}7^{5}11^{2}13^{2}17^{1}19^{1}23^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{12},\mathbf{2}>,<\mathbf{16},\mathbf{1}>]$
$T(\mathbf{16})$	[52,2,12]	$2^{49}3^{19}5^{9}7^{6}11^{2}13^{2}17^{1}19^{1}23^{1}29^{1}31^{1}$	$[<\mathbf{0},\mathbf{1}>,<\mathbf{12},\mathbf{2}>,<\mathbf{16},\mathbf{1}>]$

SRGs constructed from codes

Table: SRGs from codes spanned by fixed parts of orbit matrices for Z_2, the case with two intersections of codewords

(v, k, λ, μ)	$\lvert \mathrm{Aut}(G) \rvert$	From triangular graphs $T(2k)$
$(28, 12, 6, 4)$	8!	$5 \leq k \leq 8$
$(35, 16, 6, 8)$	8!	$5 \leq k \leq 8$
$(45, 16, 8, 4)$	10!	$6 \leq k \leq 8$
$(66, 20, 10, 4)$	12!	$7 \leq k \leq 8$
$(91, 24, 12, 4)$	14!	$k = 8$

Table: SRGs from codes spanned by fixed parts of orbit matrices for Z_2, the case with three intersections of codewords

(v, k, λ, μ)	$\lvert \mathrm{Aut}(G) \rvert$	From triangular graphs $T(2k)$
$(495, 238, 109, 119)$	$2^{21}3^{6}5^{2}7 \cdot 11 \cdot 17$	$7 \leq k \leq 8$

SRGs constructed from codes

Table: SRGs from codes spanned by nonfixed parts of orbit matrices for Z_2, the case with two intersections of codewords

(v, k, λ, μ)	$\lvert\mathrm{Aut}(G)\rvert$	From triangular graphs $T(2k)$
$(10, 3, 0, 1)$	5!	$k = 5, 8$
$(15, 8, 4, 4)$	6!	$7 \leq k \leq 8$
$(21, 10, 5, 4)$	7!	$k = 7$
$(28, 12, 6, 4)$	8!	$k = 8$
$(35, 16, 6, 8)$	8!	$k = 8$

Table: SRGs from codes spanned by nonfixed parts of orbit matrices for Z_2, the case with three intersections of codewords

(v, k, λ, μ)	$\lvert\mathrm{Aut}(G)\rvert$	From triangular graphs $T(2k)$
$(15, 8, 4, 4)$	6!	$k = 7$
$(35, 16, 6, 8)$	8!	$k = 7$

SRGs constructed from codes

Table: SRGs from codes spanned by fixed parts of orbit matrices for Z_4, the case with two intersections of codewords

(v, k, λ, μ)	$\lvert\mathrm{Aut}(G)\rvert$	From triangular graphs $T(2k)$
$(28, 12, 6, 4)$	8!	$6 \leq k \leq 8$
$(35, 16, 6, 8)$	8!	$6 \leq k \leq 8$
$(45, 16, 8, 4)$	10!	$7 \leq k \leq 8$
$(66, 20, 10, 4)$	12!	$k = 8$

Table: SRGs from codes spanned by fixed parts of orbit matrices for Z_4, the case with three intersections of codewords

(v, k, λ, μ)	$\lvert\mathrm{Aut}(G)\rvert$	From triangular graphs $T(2k)$
$(495, 238, 109, 119)$	$2^{21}3^{6}5^{2}7 \cdot 11 \cdot 17$	$k = 8$

Table: SRGs from codes spanned by parts of orbit matrices for Z_4 corresponding to orbits of length 2, the case with two intersections of codewords

(v, k, λ, μ)	$\lvert\mathrm{Aut}(G)\rvert$	From triangular graphs $T(2k)$
$(10, 3, 0, 1)$	5!	$k = 7$

An incidence structure $\mathcal{D} = (\mathcal{P}, \mathcal{B}, I)$, with point set $\mathcal{P}$, block set $\mathcal{B}$ and incidence $I \subseteq \mathcal{P} \times \mathcal{B}$, is a 2-$(v, b, r, k, \lambda)$ *design*, if $|\mathcal{P}| = v$, $|\mathcal{B}| = b$, every block $B \in \mathcal{B}$ is incident with precisely k points, every 2 distinct points are together incident with precisely λ blocks and every point is incident with exactly r blocks. If $b < \binom{v}{k}$, then $\mathcal{D}$ is called a *balanced incomplete block design* (BIBD).

Table: BIBDs from the codes of nonfixed parts of orbit matrices for Z_2 acting on $T(12)$

2-(v, b, r, k, λ)	Simple design $\mathcal{D}$	$\lvert$Aut$(\mathcal{D})\rvert$	Aut$(\mathcal{D})$
2-$(7, 28, 16, 4, 8)$	2-(7,7,4,4,2)	168	$PSL(3,2)$
2-$(15, 30, 16, 8, 8)$	2-(15,15,8,8,4)	20160	A_8
2-$(10, 30, 18, 6, 10)$	2-(10,15,9,6,5)	720	S_6

Table: BIBDs from the codes of fixed parts of orbit matrices for Z_4 acting on $T(10)$ and $T(14)$

2-(v, b, r, k, λ)	$\lvert$Aut$(\mathcal{D})\rvert$	Aut$(\mathcal{D})$
2-$(15, 15, 8, 8, 4)$	20160	A_8

THANKS FOR YOUR ATTENTION!