On orders of vertex-stabilizers in arc-transitive graphs

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Joint work with Primož Potočnik and Gabriel Verret

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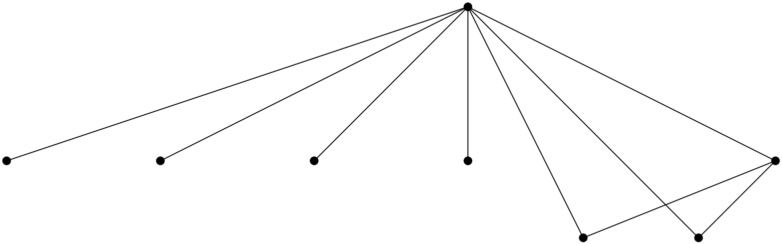

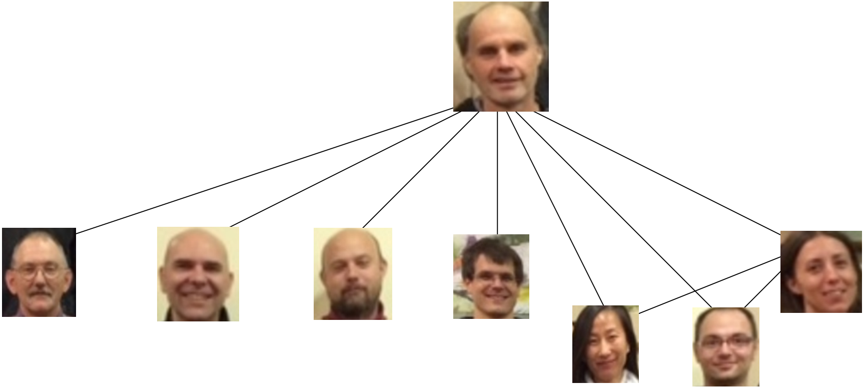

Dragan Marušič and his PhD students

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A graph Γ is said to be G-vertex-transitive, G-edge-transitive or G-arc-transitive if $G \leq \mathrm{Aut}(\Gamma)$ acts transitively on $V(\Gamma)$, $E(\Gamma)$ or $A(\Gamma)$, respectively.

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If Γ is G-vertex-transitive, then $|G| = |V(\Gamma)| \cdot |G_v|$.

Cubic arc-transitive graphs

A graph is said to be cubic if each vertex is incident with three edges.

Theorem (Tutte, 1946)

Let Γ be a connected cubic (3-regular) G-arc-transitive graph, and let $v \in V(\Gamma)$. Then

- $|G_v| = 3$ *and* G *acts regularly on the set of arcs of* Γ*;*
- $|G_v| = 6$ *and* G *acts regularly on the set of 2-arcs of* Γ*;*
- $|G_v| = 12$ *and* G *acts regularly on the set of 3-arcs of* Γ*;*
- $|G_v| = 24$ *and* G *acts regularly on the set of 4-arcs of* Γ*;*
- $|G_v| = 48$ *and* G *acts regularly on the set of 5-arcs of* Γ*.*

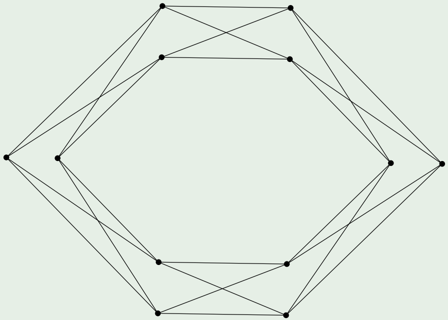
For a non-negative integer s, an *s-arc* in a graph Γ is an $(s+1)$-tuple of vertices $v_0, v_1, \ldots, v_s$, such that v_{i-1} is adjacent with v_i (for $1 \leq i \leq s$) and $v_{i-1} \neq v_{i+1}$.

Lexicographic product of graphs

The *lexicographic product* of two graphs G and H is the graph $G[H]$ with vertex set $V(G) \times V(H)$, where two vertices (x_1, y_1) and (x_2, y_2) are adjacent if and only if either $\{x_1, x_2\} \in E(G)$ or $x_1 = x_2$ and $\{y_1, y_2\} \in E(H)$.

Example

$C_6[\overline{K_2}]$



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$\mathrm{Aut}(\Gamma) = \mathbb{Z}_2 \wr D_{2n} \cong \mathbb{Z}_2^n \rtimes D_{2n}$ (for $n \geq 5$).

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It follows from the work of Gardiner (1973) that for a connected 4-valent, G-arc-transitive graph, $|G_v| \leq 2^4 3^6$, unless the action of G_v on the neighbours of v is isomorphic to D_8.

Local action

Let Γ be a connected G-vertex-transitive graph and $v \in V(\Gamma)$. Let $\Gamma(v)$ denote the neighbourhood of v in Γ. The *local action* $G_v^{\Gamma(v)}$ of G is the permutation group induced by the action of G_v on Γ_v. If Γ is a G-arc-transitive graph and L is a permutation group which is permutation isomorphic to $G_v^{\Gamma(v)}$, then we say that the pair (Γ, G) is *locally-L*.

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Definition

A transitive permutation group L is called graph-restrictive if there exists a constant $c(L)$ such that, for every locally-L pair (Γ, G) and for every arc $v \in V(\Gamma)$, the inequality $|G_v| \leq c(L)$ holds.

Examples of graph restrictive groups

Every regular group is graph restrictive, namely, if $G_v^{\Gamma(v)}$ is regular, then the stabilizer of an arc fixes all vertices incident with it, and connectedness implies that $|G_v|$ equals the valency of the graph.

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Examples of graph restrictive groups

Theorem (Trofimov and Weiss)

2-transitive groups and groups of prime degree are graph-restrictive.

Theorem (Sami, 2006)

Group D_{2k} for k odd is graph-restrictive.

Theorem (Verret, 2009)

Groups L such that $L = \langle L_x, L_y \rangle$ and L_x induces $\mathbb{Z}_p$ on y^{L_x} for some prime p (p-subregular).

Theorem (Trofimov and Weiss, 2010)

Primitive groups of linear type (primitive groups admitting a normal subgroup isomorphic to PSL in its natural action) are graph-restrictive.

A permutation group $L \leq Sym(\Omega)$ is said to be primitive, if it is transitive and preserves no non-trivial partition of Ω.

Conjecture (Weiss, 1978)

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S_5 acting on the 10 unordered pairs of a 5-set (degree 10)?

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A quasiprimitive group L is graph-restrictive if and only if there exists a constant $c'(L)$ such that, for every locally-L pair (Γ, G) with G quasiprimitive or biquasiprimitive and the inequality $|G_v| \leq c'(L)$ holds.

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Groups which are not graph restrictive

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If $L = D_8$, for which we have seen that for every positive integer n, there exists a graph Γ of order $2n$ and a group G, such that (Γ, G) is locally D_8 and $|G_v| = 2^n$.

If L is primitive or regular, then it is graph restrictive.

Transitive groups of degree 6

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Suppose that L admits a system of imprimitivity consisting of two blocks of size 3 and let A be one of these blocks. If the pointwise stabilizer $L_{(A)}$ is trivial, then L is permutationaly isomorphic to D_{12}. Order of vertex-stabilizer in this case cannot be bounded by a constant, but can be bounded by polynomial function of the order of the graph, so it is of polynomial type. In other cases with 2 blocks of size 3 it is of exponential type.

Degree 6: blocks of size 2

There are 5 imprimitive groups of degree 6 with no blocks of size 3, they are all contained in $S_2 \wr S_3$ in its imprimitive action on 6 points, and are denoted by $A_4(6)$, $2A_4(6)$, $S_4(6d)$, $S_4(6c)$ and $2S_4(6)$.

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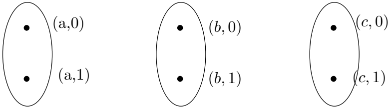

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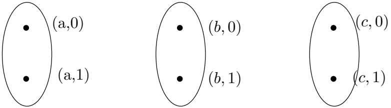

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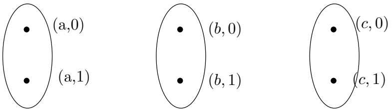

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Group $2A_4(6)$ is isomorphic to $S_2 \wr A_3$ and group $2S_4(6)$ is isomorphic to $S_2 \wr S_3$, so they are both of exponential type.

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The construction

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Similarly, $\mathbb{F}_2^{V(\Gamma)}$ also has a natural action as a group of automorphisms of Δ (given by $(v, i)^x = (v, i + x(v))$, for $x \in \mathbb{F}_2^{V(\Gamma)}$ and $(v, i) \in V(\Gamma) \times \mathbb{F}_2$).

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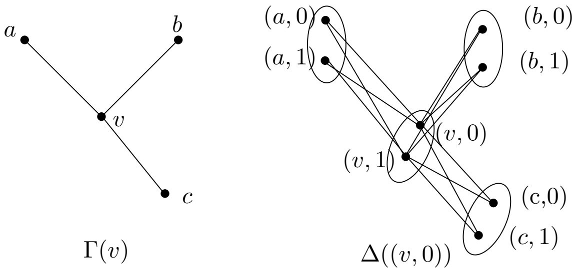

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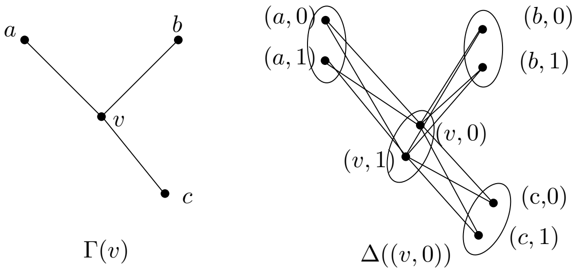

Recall that the kernel in the local action is $\mathbb{Z}_2^2$.

Let E_1 be the 1-eigenspace for Γ over $\mathbb{F}_2$, that is

$$E_1 = \{x \in \mathbb{F}_2^{V(\Gamma)} : x(v) + \sum_{a \in \Gamma(v)} x(a) = 0 \ (\forall v \in V(\Gamma)\}.$$

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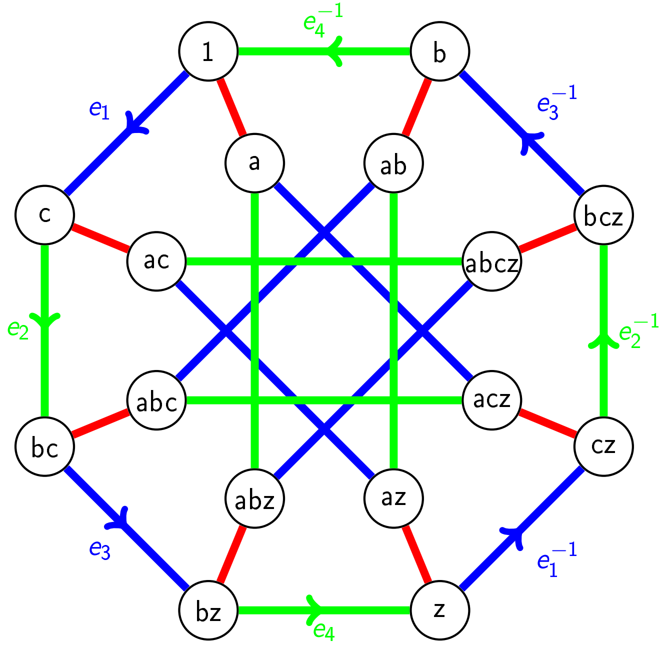
How to find cubic arc-transitive graphs with large 1-eigenspaces?

Möbius–Kantor graph

$$R = \langle a, b, c, z \mid 1 = a^2 = b^2 = c^2 = z^2 = [a, z] = [b, z] = [c, z], \\ [a, b] = [b, c] = [a, c] = z\rangle.$$

The Cayley graph $\mathcal{M} = \mathrm{Cay}(R, \{a, b, c\})$ is called Möbius–Kantor graph.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az
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e_1
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e_3
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Automorphisms of Möbius–Kantor graph

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Note that B is 2-arc-regular and contains an arc-regular subgroup A of the form $R \rtimes \mathbb{Z}_3$.

$\mathbb{Z}_n^4$ covers of Möbius–Kantor graph

Let n be a positive integer and let $N = \mathbb{Z}_n^4 = \langle e_1, e_2, e_3, e_4 \rangle$. Let $\mathcal{M}_n$ be the derived covering graph of $\mathcal{M}$ (with respect to the voltage assignment given in the previous figure).

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These graphs were studied by Malnič, Marušič, Miklavič and Potočnik in 2007. It follows from their work that B lifts to $\mathcal{M}_n$. We will denote its lift by $\tilde{B}$. Similarly, let $\tilde{A}$ be the lift of A. Note that $\tilde{B}$ is 2-arc-regular on $\mathcal{M}_n$, and $\tilde{A}$ is 1-arc-regular.

Theorem (H., Potočnik, Verret)

Let $n \geq 3$ be a positive integer and let E_1 be the 1-eigenspace for $\mathcal{M}_n$ over $\mathbb{F}_2$. Then $|E_1| \geq 2^{|V(\mathcal{M}_n)|/72}$.

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$$G_1 := \langle E_1, \tilde{A} \rangle = E_1 \rtimes \tilde{A} \leq \mathrm{Aut}(\Delta)$$
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(Δ, G_1) is locally $A_4(6)$ and (Δ, G_2) is locally $S_4(6d)$.

Theorem (H., Potočnik, Verret)

Let $n \geq 3$ be a positive integer and let E_1 be the 1-eigenspace for $\mathcal{M}_n$ over $\mathbb{F}_2$. Then $|E_1| \geq 2^{|V(\mathcal{M}_n)|/72}$.

Remark

Computational data suggests that $|E_1| = 2^{|V(\mathcal{M}_n)|/8+2}$ if n is odd, and $|E_1| = 2^{|V(\mathcal{M}_n)|/8+8}$ if n is even.

$$G_1 := \langle E_1, \tilde{A} \rangle = E_1 \rtimes \tilde{A} \leq \mathrm{Aut}(\Delta)$$
$$G_2 := \langle E_1, \tilde{B} \rangle = E_1 \rtimes \tilde{B} \leq \mathrm{Aut}(\Delta)$$

(Δ, G_1) is locally $A_4(6)$ and (Δ, G_2) is locally $S_4(6d)$.

$S_4(6c)$ is solved by considering $\langle G_1, \tau\sigma \rangle$ where $\tau \in \tilde{B} \setminus \tilde{A}$ and $\sigma \in \mathbb{F}_2^{V(\Gamma)}$ with some additional properties.

Open problems

Problem

Determine the growth of vertex-stabilizers in 8-valent arc-transitive graphs.

Problem

Does there a transitive group that is neither graph-restrictive nor of polynomial or exponential type.

Thank you!!!