

A COARSE ERDŐS-PÓSA THEOREM

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Abstract

An induced packing of cycles in a graph is a set of vertex-disjoint cycles with no edges between them. We generalise the classic Erdős–Pósa theorem to induced packings of cycles. More specifically, we show that there exists a function $f(k) = O(k \log k)$ such that for every positive integer k , every graph G contains either an induced packing of k cycles or a set X of at most $f(k)$ vertices such that the closed neighbourhood of X intersects all cycles in G . Our proof is constructive and yields a polynomial-time algorithm finding either the induced packing of cycles or the set X .

As a corollary, we prove that every graph with no $K_{1,t}$ induced subgraph and no induced packing of k cycles has tree-independence number at most $O(tk \log k)$, and one can construct a corresponding tree-decomposition in polynomial time. This resolves a special case of a conjecture of Dallard et al. (arXiv:2402.11222), and implies that on such graphs, many NP-hard problems, such as Maximum Weight Independent Set, Maximum Weight Induced Matching, Graph Homomorphism, and Minimum Weight Feedback Vertex Set, are solvable in polynomial time.

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