# RECENT DISCOVERIES ABOUT FINITE QUOTIENTS OF TRIANGLE GROUPS

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## Abstract

For integers $k, l, m > 1$, the ordinary *triangle group* $\Delta^+(k, l, m)$ is the group with presentation $\langle\, x, y \mid x^k = y^l = (xy)^m = 1 \,\rangle$, or equivalently $\langle\, x, y, z \mid x^k = y^l = z^m = xyz = 1 \,\rangle$. Such groups play an important role in the study of large automorphism groups of algebraic curves and compact Riemann surfaces, and of regular maps on orientable and non-orientable surfaces. In this talk I will describe some recent developments concerning their finite quotients:

(a) a surprising discovery that despite decades of attention being paid to non-abelian simple quotients, such quotients are relatively rare;

(b) some work with my PhD student Darius Young why non-perfect hyperbolic ordinary triangle groups have vast numbers of finite soluble quotients;

(c) a new theorem by Darius Young showing that the natural density (in the positive integers) of the set of orders of finite quotients of every given triangle group is **zero**, proved without resorting to the classification of finite simple groups; and

(d) some unexpected consequences for the densities of orders of finite edge-transitive graphs, and the density of orders of finite quotients of any given finitely-generated infinite group.

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