LINEAR COMPLETE SYMMETRIC RANK-DISTANCE CODES IN $M_{3\times 3}(\mathbb{F}_q)$

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Abstract

An $\mathbb{F}_q$-linear code of minimum distance d is said to be *complete* if it is not contained in any larger $\mathbb{F}_q$-linear code with the same minimum distance. Studying such extremal objects is a natural problem in coding theory.

In this talk, we classify $\mathbb{F}_q$-linear complete symmetric rank-distance (CSRD) codes in $M_{3\times 3}(\mathbb{F}_q)$ up to equivalence. In particular, we show that there exist exactly 3 such codes when q is odd, and 6 when q is even of minimum distance 2, up to equivalence.

This classification reveals a fundamental distinction between odd and even characteristic. When q is odd, every CSRD code in $M_{3\times 3}(\mathbb{F}_q)$ is in fact a maximum symmetric rank-distance (MSRD) code. In contrast, when q is even, there exist 3-dimensional CSRD codes that are not MSRD.

Our approach is geometric. By interpreting symmetric rank-distance codes as subspaces of $\mathrm{PG}(5, q)$, the problem translates into the study of linear systems of conics in $\mathrm{PG}(2, q)$, in particular pencils and nets of conics corresponding to 1- and 2-dimensional subspaces of the projective space of quadratic forms in $\mathbb{F}_q[X, Y, Z]$.

We conclude by discussing consequences of this correspondence and provide new insight toward the long-standing open problem of classifying and characterizing nets of conics in $\mathrm{PG}(2, q)$.

Discrete Mathematics Day on the Adriatic Coast
InnoRenew CoE, Izola, Slovenia
May 12, 2026