12th PhD Summer School in DISCRETE MATHEMATICS

Koper, Slovenia

7-13 September, 2025

Book of Abstracts

Supported by ARIS (Slovenian Research and Innovation Agency) and MVZI (Ministry of Higher Education, Science and Innovation).

Welcome

Dear Colleague!

What was started in 2010 as an informal research collaboration has now grown into a colorful series of international workshops and summer schools. We are glad to see many participants returning and several new ones joining the creative atmosphere of this event, which we will try to keep as relaxed and uplifting as in previous years. The organization of the meeting comes as a combined effort of the Faculty of Mathematics, Natural Sciences and Information Technologies (UP FAMNIT), the Andrej Marušič Institute (UP IAM), two members of the University of Primorska, and the Slovenian Discrete and Applied Mathematics Society, and is in line with our goal to create an international research center in algebraic combinatorics in this part of the world.

We wish you a pleasant and mathematically fruitful week at Koper.

Scientific Committee *(Ademir Hujdurović, Klavdija Kutnar, Aleksander Malnič, Dragan Marušič, Štefko Miklavič, Primož Šparl)*

12th PhD
Summer School in
Discrete Mathematics
Koper, Slovenia, 7-13 September 2025

General Information

12th PhD Summer School in Discrete Mathematics

UP FAMNIT, Koper, Slovenia, June 25 – July 1, 2023.

Organized by

• UP FAMNIT *(University of Primorska, Faculty of Mathematics, Natural Sciences and Information Technologies);*
• UP IAM *(University of Primorska, Andrej Marušič Institute);*
• Slovenian Discrete and Applied Mathematics Society.

In Collaboration with

• IMFM - Institute of Mathematics Physics and Mechanics;
• Centre for Discrete Mathematics, UL PeF *(University of Ljubljana, Faculty of Education)*.

PhD Summer School in Discrete Mathematics Minicourses:
A short course in spectral graph theory and distance-regular graphs
Sebastian Cioaba, *University of Delaware, USA*
Container method in combinatorics
Rajko Nenadov, *University of Auckland, New Zealand*

Scientific Committee:
Ademir Hujdurović, Klavdija Kutnar, Aleksander Malnič, Dragan Marušič, Štefko Miklavič, Primož Šparl

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MINICOURSE DESCRIPTIONS

A short course in spectral graph theory and distance-regular graphs

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In this short course, I will present the basics of spectral graph theory and distance-regular graphs. The presentation will be self-contained and after a short recap of linear algebra, we will describe the most common used matrices in spectral graph theory (adjacency matrix, Laplacian, signed adjacency matrix, normalized Laplacian, signless Laplacian), their eigenvalues and their basic properties. We will present some older and some recent applications of eigenvalues and spectral methods. For distance-regular graphs, we will describe the basic combinatorial and linear algebraic theory and some connections between distance-regular graphs and approximation algorithms in theoretical computer science and the problem of embedding graphs into Euclidean spaces with least distortion.

Container method in combinatorics

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The container method is a powerful technique for enumerating independent sets in graphs and hypergraphs enjoying a sufficiently uniform edge distribution. Very briefly, the method exploits a clustering phenomenon exhibited by independent sets in such hypergraphs. These (hyper)graphs often arise in practice, making the container method widely applicable. The method frequently provides additional structural properties of independent sets, which are crucial for some applications.

Graph containers were pioneered by Kleitman and Winston in their seminal work on enumerating graphs without a cycle of length 4. The method was further refined by several researchers, most notably by Sapozhenko in the context of enumerating independent sets in regular graphs and sum-free sets. This line of research culminated in the work of Balogh, Morris, and Samotij, and, independently, Saxton and Thomason, who generalised the method to hypergraphs.

We begin with the basic graph container method and discuss classic applications, such as counting C4-free graphs and sum-free sets. We then move on to hypergraph containers and count the number of triangle-free graphs. From there, we explore numerous surprising applications of the hypergraph containers. These include the typical structure of triangle-free graphs, the Ramsey theorem for random graphs, and a construction of points in the Euclidean plane with no four on a line, where every sufficiently large set contains a collinear triple. We conclude this series of lectures by going over a recent short proof of hypergraph containers by Nenadov and Pham.

Invited Speakers

The linear dimension of permutation groups

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From a linear group G acting on a vector space V we obtain transitive permutation representations of G, the orbits on vectors. One may ask to what extent these permutation representations determine the linear representation. Conversely, given a transitive permutation group G, we may ask which linear representations of G yield an orbit of G on vectors that is equivalent to the permutation action of G. The 'linear dimension' of a permutation group G is the smallest dimension of such a linear representation. One can see this as the most efficient way to linearise the action of G. This was defined recently by Dâ€™Alconzo and Di Scala and is inspired by cryptographic questions. In this talk I will give some overview of the recent results and explain how this leads to fundamental and difficult questions in the representation theory of finite groups.

Eigenvalue bounds for the independence number of graph powers and their application to coding theory

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Spectral graph theory looks at the connection between the eigenvalues of a matrix associated with a graph and the corresponding structures of a graph. In this talk we will show how spectral methods provide a handy tool for obtaining results concerning the structure of graphs, and also how these results are a powerful tool to be used in other fields. In particular, we will derive sharp eigenvalue bounds on an NP-hard distance-type graph parameter: the k-independence number of a graph (or equivalently, the independence number of a graph power). We will see how to use polynomials and mixed integer linear programming in order to optimize such bounds. Finally, we will illustrate some applications of the new eigenvalue bounds to coding theory.

New directions for direction sets

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Let q be a power of a prime p, and let f be a function from $GF(q)^m$ to $GF(q)$. The graph of f is the set of points in the affine space $AG(m+1,q)$ of the form $(x, f(x))$, where x ranges over $GF(q)^m$. The directions determined by the graph of f are the points at infinity corresponding to the slopes of lines connecting pairs of points on the graph.

In this talk, we will show how properties of the set of determined directions yield information about the function. We will explain how the graphs of functions and their associated direction sets can be used to construct various extremal objects in finite geometry, for instance, the smallest point sets in the plane that meet every line in at least t points, or the smallest multisets in space that meet each line in a number of points divisible by p. We will also discuss applications related to an Erdös-Ko-Rado type problem.

Quasirandomness through lenses of combinatorial limits

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A combinatorial object is said to be quasirandom if it resembles a random object in a certain robust sense. The notion of quasirandom graphs, which was developed in the work of Rödl, Thomason, Chung, Graham and Wilson in 1980s, is particularly robust as several different properties of truly random graphs, e.g., subgraph density, edge distribution and spectral properties, are satisfied by a large graph if and only if one of them is.

We will present classical and recent results on quasirandomness of different combinatorial objects, in particular, graphs, directed graphs, permutations, hypergraphs and Latin squares. We will cast the results using the language of the theory of combinatorial limits, which we introduce during the talk, and demonstrate how analytic methods provided by the theory of combinatorial limits can be used to obtain results concerning quasirandom combinatorial objects.

Factored lifts of graphs and their spectra: The Abelian and non-Abelian cases

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In this talk, we introduce the concept of factored lift, associated with a combined voltage graph, as a generalization of the lift graph (or voltage graph). We present a new method for computing all the eigenvalues and eigenspaces of factored lifts in two cases, when the underlying group is Abelian and non-Abelian.

Best Student Talk Award Recipient
Monday, 8 September

Prime Simplicial Complexes of Finite Groups

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In 2024, Peter Cameron proposed generalisations of various graphs defined on groups to simplicial complexes. I will talk about one of these, the prime simplicial complex $\Pi(G)$ of a finite group G, which is composed of all sets of primes S where G has an element of order the product of primes in S, with the subsets partially ordered by inclusion. This is a generalization of the well-studied prime (or Gruenberg-Kegel) graphs. I will present new results about recognizability by prime simplicial complex, and the purity of the prime simplicial complex, a question asked by Peter Cameron.

Joint work with Melissa Lee.

Student Talks – Monday, 8 September

Cubic girth-regular graphs of girth six

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Recall that given a graph Γ, the girth of Γ is the length of a shortest cycle in Γ. In graphs of finite girth, each vertex v of Γ can be associated with a non-decreasing sequence of integers, one integer for each edge incident with v, representing the number of girth cycles containing that edge (possibly 0). If the sequences associated with the vertices of Γ are all identical (and so Γ is regular), we say that Γ is *girth-regular,* and the shared sequence is said to be the *signature of* Γ. Note that all vertex-transitive graphs are necessarily girth-regular.

The concept of girth-regularity was introduced in [1]. There, several necessary conditions on signatures of cubic girth-regular graphs were proved, together with a classification of all cubic girth-regular graphs of girth at most 5. Consequently, in [2] all cubic vertex-transitive graphs of girth 6 were classified according to their signatures. In our work, we extend the latter to a characterization of all cubic girth-regular graphs of girth 6. We note that for this girth there exists a vertex-transitive graph for each of the attainable signatures. In addition, inspired by the arguments for girth 6, we prove multiple additional conditions on signatures of cubic girth-regular graphs of any even girth.

This is joint work with Robert Jajcay, Maruša Lekše and Primož Potočnik.

Acknowledgments. The work has been supported by grants VEGA 1/0437/23 and UK/1398/2025.

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Graph energy through the lenses of semidefinite programming

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Let G be a graph on n vertices with independence number $\alpha(G)$. Let A be the adjacency matrix of G with eigenvalues $\lambda_1 \geq \cdots \geq \lambda_n$. Let $\sum_{\lambda_i > 0} \lambda_i = \frac{1}{2}\mathscr{E}$, where $\mathscr{E}$ is the energy of a graph. Using Graffiti, Fajtlowicz proposed the following conjecture.

Conjecture 1. [1] $\frac{1}{2}\mathscr{E} \geq n - \alpha(G)$.

Let $\chi(G)$ be the chromatic number of G. The closest known result to Conjecture 1 is due to Hoffman.

Theorem 2. [2] $\frac{1}{2}\mathscr{E} \geq n - \chi(\overline{G})$.

Let $\chi_f(G)$ be the fractional chromatic number of G. We formulate the graph energy as a semidefinite program and give a step towards Conjecture 1, showing the following result.

Theorem 3. $\frac{1}{2}\mathscr{E} \geq n - \chi_f(\overline{G})$.

Using the semidefinite program formulation, we also relate the graph energy with the well-known ratio bound for the independence number. Moreover, we give stronger results for the class of highly regular graphs, a generalization of distance-regular graphs.

This is joint work with Aida Abiad, Gabriel Coutinho and Luuk Reijnders.

Acknowledgments. The work has been supported by FAPEMIG through the grant APQ-04481-22.

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Classification of unstable circulant graphs of twice odd order

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A canonical double cover of a simple graph $\Gamma = (V, E)$ is the tensor product $\Gamma \times \mathrm{K}_2$. A graph is called stable when the automorphism group of this double cover is as small as possible. More precisely, a graph is stable when $\mathrm{Aut}(\Gamma \times \mathrm{K}_2) \cong \mathrm{Aut}(\Gamma) \times \mathrm{Aut}(\mathrm{K}_2)$ and unstable otherwise. The problem of stability proved to be closely connected with studying symmetries of more general tensor products (cf. [1]). The focus was put on classifying unstable graphs in certain well structured graph families such as circulants.

In two previous years Hujdurović and Kovács used theory of Schur rings to obtain interesting partial result [2]. I expand on those methods by mixing them with classification of primitive group actions containing cyclic regular action and the theory of Galois cohomology, finally obtaining the following.

Theorem 1. *Let $m > 1$ be an odd integer and let $\Gamma = Cay(\mathbb{Z}_{2m}, S)$ be a connected graph. Then Γ is unstable if and only if*

(i) there exists nonzero $h \in 2\mathbb{Z}_{2m}$ such that $S \cap 2\mathbb{Z}_{2m} + h = S \cap 2\mathbb{Z}_{2m}$;

(ii) or $Cay(\mathbb{Z}_{2m}, S) \cong Cay(\mathbb{Z}_{2m}, S + m)$.

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Cyclic m-DCI-groups and m-CI-groups

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Based on the earlier work of Li [3] from 1997 and Dobson [1] from 2008, in this talk, we present the classification of cyclic m-DCI-groups and m-CI-groups.

Theorem 1. [2] *For a positive integer m such that $m \geq 3$, the group $\mathbb{Z}_n$ is an m-DCI-group if and only if n is not divisible by 8 nor by p^2 for any odd prime $p < m$. Furthermore, if $m \geq 6$, then $\mathbb{Z}_n$ is an m-CI-group if and only if either $n \in \{8, 9, 18\}$, or $n \notin \{8, 9, 18\}$ and n is not divisible by 8 nor by p^2 for any odd prime $p < \frac{m-1}{2}$.*

This is joint work with István Kovács.

Acknowledgments. The work has been supported by the Slovenian Research Agency (research program P1-0285) and Young Researchers Grant

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Extremal problem for graphs with modular p-group symmetry

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For a finite group G, define $\alpha(G)$ as the minimum number of vertices among all graphs Γ such that $\mathrm{Aut}\,\Gamma \cong G$ [1]. For any p prime, all p-groups of order p^n having cyclic subgroups of order $p^n - 1$ have been completely classified. Several authors have already investigated some of these families of groups in order to find vertex-minimal graphs [2]. Here we consider a family of groups called modular p-groups, for an odd prime p and $n \geq 3$. A modular p-group is defined as $\mathrm{Mod}_n(p) = \langle a^{p^{n-1}} = 1, b^p = 1, ba = a^{p^{n-2}+1}b \rangle$. We compute the order of vertex-minimal graphs with $\mathrm{Mod}_n(p)$-symmetry. The fixing number of a graph Γ is defined as the smallest number of vertices in $V(\Gamma)$ that, when fixed, makes $\mathrm{Aut}\,\Gamma$ trivial. This concept has been extended to finite groups by Gibbons and Laison [3]. For a finite group G, the fixing set is defined as the set of all fixing numbers of graphs having automorphism groups isomorphic to G. We show that any graph Γ whose automorphism group is a modular p-group has the fixing number 1. As a result, the modular p-group's fixing set becomes $\{1\}$.

Theorem 1.[4] *Let p be odd prime and $n \geq 3$ be an integer. Then* $\alpha(\mathrm{Mod}_n(p)) = p^n$.

Using the theory of transitive permutation groups, we discussed the cycle decomposition of the generators of modular p-group and then evaluated the fixing set.

Theorem 2. [4] *Let p be odd prime and $n \geq 3$ be an integer. Then* $\mathrm{fix}(\mathrm{Mod}_n(p)) = \{1\}$.

Example. Let $G = \mathrm{Mod}_3(3)$ and $S = \{\beta, \gamma, \beta^8, \gamma^2, \beta\gamma, \gamma^2\beta^8\} \subset G$. The Cayley graph $Cay(G,S)$ has automorphism group isomorphic to G with 27 vertices and the edge set $E(Cay(G,S))$, that comprises the edge orbits $O\{\beta,\beta^2\}$, $O\{\beta,\beta\gamma\}$, $O\{\beta,\beta^2\gamma\}$, $O\{\beta,\beta\gamma^2\}$, and $O\{\beta,\beta^3\gamma^2\}$.

This is joint work with Dr. Ranjit Mehatari

Acknowledgments. The work has been supported by CSIR Grant No- 09/0983(16243)/2023-EMR-I.

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Student Talks – Tuesday, 9 September

Eigenvalue bounds for the distance-t chromatic number of a graph and their application to Lee codes

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In this talk we derive eigenvalue bounds for the distance-t chromatic number of a graph, which is defined as the fewest colors needed to color all the vertices of a graph such that no vertices within distance t share a color. We apply such bounds to hypercube graphs, providing alternative spectral proofs for results by Ngo, Du and Graham [1], and improving their bound for several instances. We also apply the eigenvalue bounds to Lee graphs, extending results by Kim and Kim [2]. Finally, we provide a complete characterization for the existence of perfect Lee codes of minimum distance 3. In order to prove our results, we use a mix of spectral and number theory tools. Our results, which provide the first application of spectral methods to Lee codes, illustrate that such methods succeed to capture the nature of the Lee metric.

This is joint work with Aida Abiad and Alessandro Neri.

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Counting cospectral graphs made via switching

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‘Are almost all graphs determined by their spectrum¿ is one of the main open questions in spectral graph theory. Haemers‘ conjectured that the answer is yes. A graph is determined by their spectrum if there is no non-isomorphic graph with the same spectrum (a cospectral mate). The main way to construct cospectral mates is through switching methods. We have studied these switching methods and found a general framework to count how often a switching method creates a cospectral mate. This gives a formula depending on certain parameters of the switching method. **Theorem 1.** *Let Q be an $m \times m$ orthogonal matrix such that (Q,Γ)-switching is distinguishing and produces at least one cospectral mate. Then there are*

$$\frac{1}{|Aut_Q(\Gamma)|}|\mathcal{V}_Q|^{n-m}g_{n-m}(1+o(1)) \tag{1}$$

graphs on n vertices that have a cospectral mate through (Q,Γ)-switching. In this talk I will explain switching methods and this formula and highlight some of the challenges we might face in applying our formula to attack Haemers conjecture. This talk is based on [1].

This is joint work with Aida Abiad and Robin Simoens.

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The Hoffman bound number; a shift of perspective Hoffman colorings of (strongly) regular graphs

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Hoffman's bound is a well-known lower bound on the chromatic number of a graph in terms of the eigenvalues of the adjacency matrix of the graph. We view this spectral bound as a parameter itself instead of a bound, and call it the Hoffman number accordingly. This shift of perspective leads to multiple applications of colorings attaining the Hoffman bound (Hoffman colorings) for several notions of graph regularity: regular, (co-)edge-regular, and strongly regular.

For strongly regular graphs, we relate the Hoffman number to the notion of pseudo-geometricity:

Theorem 1. *A strongly regular graph is pseudo-geometric if and only if its Hoffman number is integral. In particular, Hoffman colorable strongly graphs are pseudo-geometric.*

Moreover, we show the following characterization of strongly regular graphs among regular graphs in terms of the Hoffman number:

Theorem 2. *Let G be a regular graph with Hoffman number h, and let $\overline{h}$ be the Hoffman number of the complement of G. Then G is strongly regular if and only if $h \cdot \overline{h} = |V(G)|$.*

This has the following consequence:

Corollary 1. *A co-edge-regular graph that is not strongly regular cannot be Hoffman colorable.*

The Hoffman number is therefore an interesting parameter to consider in (strongly) regular graphs.

This is joint work with Aida Abiad and Bart De Bruyn.

Algebraic bounds for sum-rank-metric codes

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The sum-rank metric is a generalization of the well-known Hamming and rank metrics. In this talk, we introduce two new bounds on the maximal cardinality of the sum-rank-metric code with a given minimum distance. One of the bounds exploits a connection between such a code and a $(d-1)$-independent set in a graph defined for the sum-rank-metric space. We then use the eigenvalues of the graph to deduce the bound. The second bound is derived from the Delsarte's LP method, which has been previously obtained for Hamming metric, rank metric, Lee metric, and others, but the sum-rank-metric case remained open. To derive the new LP bound, we propose a way to construct an association scheme for the sum-rank metric, since the approach used in the Hamming and the rank-metric cases fails due to the associated graph not being distance-regular in general. Based on computational experiments on relatively small instances, we observe that the obtained bounds often outperform the bounds previously known for sum-rank-metric codes.

The talk is based on [1, 2].

This is joint work with Aida Abiad, Alexander Gavrilyuk, Ilia Ponomarenko, and Alberto Ravagnani.

Acknowledgments. Antonina P. Khramova is supported by NWO via the grant OCENW.KLEIN.475.

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On co-edge-regular graphs with four distinct eigenvalues

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A connected graph is called co-edge-regular with parameters (v,k,c) if it is k-regular and any two distinct vertices have exactly c common neighbors. Tan-Koolen-Xia [1] conjectured that connected co-edge-regular graphs with exactly four distinct eigenvalues and fixed smallest eigenvalue, when having sufficiently large valency, belong to two different families of graphs.

In this talk, we focus on co-edge-regular graphs with exactly four distinct eigenvalues. In [2], we present some counterexamples of Tan-Koolen-Xia conjecture, demonstrating that this class of graphs exhibits far more diverse behavior than previously expected. In particular, we show that even co-edge-regular graphs that are cospectral with the 2-clique extension of a Latin square graph with smallest eigenvalue −3 are not determined by their spectrum.

This is joint work with Jack H. Koolen.

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Signed lollipops and their adjacency cospectral mates

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A lollipop is a unicyclic graph consisting of a cycle joined to the endvertex of a path. Here we consider signed lollipops and their adjacency spectra. In 2015, Belardo and Petecki shown that a signed lollipop is determined by its Laplacian spectrum, that is, a signed graph Laplacian cospectral to a signed lollipop is switching isomorphic to it. In this talk, we show that the same result does not hold for the adjacency spectrum, and we identify several classes of signed lollipop which are not determined by their adjacency spectra by providing the switching nonisomorphic cospectral mates.

This is joint work with Francesco Belardo, Maurizio Brunetti, Adriana Ciampella.

Student Talks – Wednesday, 10 September

Coloring outside the lines

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In this talk, I will talk about [1, 2], concerning spectral bounds that connect several notions of coloring numbers with the eigenvalues of the normalized Laplacian.

I will first define several notions of vertex (hyper)graph coloring and the corresponding coloring numbers. Given a notion of vertex coloring, the corresponding coloring number is the minimum number of colors needed to color the vertices of the (hyper)graph according to the definition.
Fix a hypergraph $\Gamma = (V, E)$ with vertex set $\{v_1, \dots, v_N\}$.

(i) A vertex coloring is a *proper strong (vertex) k-coloring* if any two vertices sharing an edge have different colors. The corresponding coloring number is denoted by $\chi := \chi(\Gamma)$.

(ii) If Γ is a graph, then a vertex coloring is *d-improper* if every vertex v has at most d neighbors with the same color as v. The corresponding *d-improper coloring number* of Γ is denoted by $\chi^d := \chi^d(\Gamma)$.

(iii) A (vertex) k-coloring is *d-proper* if every edge contains at most d vertices of every color. The corresponding coloring number is denoted by $\chi_d := \chi_d(\Gamma)$.

We shall then define the normalized Laplacian. To this end, we need the following definitions. The *degree* of a vertex v is defined as the number of edges containing v. The *degree matrix* of Γ is defined by $D := \operatorname{diag}(\deg(v_1), \deg(v_2), \dots, \deg(v_N))$, and the vertex-edge incidence matrix is denoted by $\mathscr{I}$. The normalized Laplacian of Γ is defined as

$$L := D^{-1}\mathscr{I}\mathscr{I}^\top.$$

The normalized Laplacian eigenvalues are denoted by $\lambda_1 \le \lambda_2 \le \dots \le \lambda_N$.

I provide bounds relating the normalized Laplacian eigenvalues to definitions (i)–(iii):

$$\chi(\Gamma) \ge \frac{\lambda_N - \lambda_1}{\min\{\lambda_N - 1, 1 - \lambda_1\}}, \tag{2}$$

$$\chi^d(\Gamma) \ge \frac{\lambda_N}{\lambda_N - 1 + d/\overline{\deg}}, \tag{3}$$

where $\overline{\deg}$ denotes the average degree of G in (3).

Assuming that our hypergraph is *c-uniform* (all of its edges have the same size c), we obtain the bound

$$\chi_d(\Gamma) \ge \frac{c - \lambda_1}{d - \lambda_1}. \tag{4}$$

Finally, for all three bounds, we give necessary conditions for hypergraphs for which the bound is an equality.

This is joint work with Raffaella Mulas.

Acknowledgments. Raffaella Mulas is supported by the Dutch Research Council (NWO) through the grant VI.Veni.232.002.

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Designs of perfect matchings

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It is well-known that the complete graph K_{2n} on $2n$ vertices can always be decomposed into perfect matchings, called a 1-factorisation. In such a decomposition, every edge of K_{2n} appears in exactly 1 perfect matching. This was generalised by Jungnickel and Vanstone to *hyperfactorisations*. These are sets of perfect matchings such that every pair of disjoint edges of K_{2n} appears in a constant number of perfect matchings. Hyperfactorisations are examples of Cameron's *partition systems* and were rediscovered by Stinson who called them *hyperresolutions*. We generalise all these ideas to λ-factorisations of K_{2n} and characterise them algebraically as Delsarte designs in an association scheme using the theory of Gelfand pairs. We use this characterisation to derive divisibility conditions and non-existence results. Furthermore, we explore a connection to finite geometry, giving rise to explicit constructions of λ-factorisations.

Joint work with John Bamberg (University of Western Australia).

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Construction of Legendre pairs from linear codes

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In this work, we study connections between Legendre pairs, 2-designs and linear codes. Starting with a given Legendre pair, we construct a corresponding 2-design and span a linear code by its incidence matrix. Further, we search for 2-designs, i.e. Legendre pairs, in the constructed codes.

This is joint work with Dean Crnković and Andrea Švob.

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The H-join of complex unit gain graphs and their stability

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Let H be a graph of order k and let $\mathscr{F} = \{G_1, G_2, \ldots, G_k\}$ be a family of graphs. Then the H-join of the family $\mathscr{F}$ is obtained by replacing each vertex v_i of H with the graph G_i of $\mathscr{F}$ and respecting the adjacencies existing in H. We present two distinct definitions of the H-join of complex unit gain graphs. The first is a direct extension of the H-join of signed graphs and is stable under switching of H. The second, on the other hand, requires the fixing of a spanning forest in order to be defined, and is also stable under switchings of the component G_i graphs. A spectral analysis is provided for both constructions.

This is joint work with Stefano Spessato and Federico Belardo.

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The extremal spectral radius of generalized block graphs

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A block graph is a graph in which every block is a complete graph. Denote by $\mathscr{K}_{n,q}$ the set of block graphs with n vertices and all blocks on $q+1$ vertices for every $q \geq 2$. Recently, Zhao and Liu [1] determined the minimum spectral radius of graphs in $\mathscr{K}_{n,q}$, which verified a conjecture posed by Conde et al. [2]. Replacing the complete graph by a general block or a cycle, we define a generalized block graph or a cycle tree, respectively. Let $\mathscr{B}_{n,q}$ (resp. $\mathscr{C}_{n,q}$) be the set of generalized block graphs (resp. cycle trees) on n vertices and in which each block is of order $q+1$ for $q \geq 2$. In this talk, we will consider the maximum/minimum spectral radius of a graph in $\mathscr{C}_{n,q}$. Furthermore, we will also discuss the extremal graph attaining the minimum spectral radius among graphs in $\mathscr{B}_{n,q}$, which coincides with that in $\mathscr{C}_{n,q}$. The main results are contained in the following two Theorems:

Theorem 1. *Let $G \in \mathscr{C}_{n,q}$ and $n = bq+1$. Then*

(i) $\rho(G) \leq \rho(CS(n,q))$ *with equality if and only if* $G = CS(n,q)$.

(ii) $\rho\left(CP_b^q\right) \leq \rho(G)$ *with equality if and only if* $G = CP_b^q$.

Theorem 2. *Let $G \in \mathscr{B}_{n,q}$ and $n = bq+1$. Then*

(i) $\rho(G) \leq \rho(KS(n,q))$ *with equality holds if and only if* $G = KS(n,q)$.

(ii) $\rho(CP_b^q) \leq \rho(G)$ *with equality holds if and only if* $G = CP_b^q$.

This is joint work with Ji-Ming Guo and Zhiwen Wang.

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Reconfigurations of Plane Caterpillars

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Given a point set S in the plane, a *plane spanning tree* on S is a spanning tree of S whose edges are straight line segments that do not cross. Let $\mathscr{T}(S)$ be the set of all plane spanning trees on S. We define the following five reconfigurations on $\mathscr{T}(S)$. For the following, we are given plane spanning trees $T_1 = (S, E_1)$, $T_2 = (S, E_2) \in \mathscr{T}(S)$, then we say that:

(i) T_1 and T_2 are connected by a *flip* if $E_2 = E_1 \setminus \{e\} \cup \{f\}$ for some edges e, f.

(ii) T_1 and T_2 are connected by a *compatible flip* if $E_2 = E_1 \setminus \{e\} \cup \{f\}$ for some edges e, f which do not cross.

(iii) T_1 and T_2 are connected by a *rotation* if $E_2 = E_1 \setminus \{e\} \cup \{f\}$ for some edges e, f which share an endpoint.

(iv) T_1 and T_2 are connected by an *empty triangle rotation* if $E_2 = E_1 \setminus \{e\} \cup \{f\}$ for some edges e, f which share an endpoint and the triangle spanned by their endpoints is empty.

(v) T_1 and T_2 are connected by a *slide* if $E_2 = E_1 \setminus \{e\} \cup \{f\}$ for some edges e, f which share an endpoint and the triangle spanned by their endpoints is empty and if $e = ab$ and $f = ac$ then $bc \in E_1 \cap E_2$.

It is known that the reconfiguration graph of plane spanning trees is connected even in the most restrictive case when the reconfigurations are slides [1]. Consequently, so are the reconfiguration graphs for all other types of reconfigurations we have defined. The same question remains unanswered for the subgraph of this reconfiguration graph induced by plane spanning paths of point sets in general position. Its connectivity has been conjectured for a long time [2], and it is known that the flip graph contains a large connected component [3].

We study a larger subclass of trees: caterpillars. A *caterpillar* is a tree in which all non-leaf vertices form a path. For a set S, we denote by $\mathscr{C}(S)$ the set of all plane spanning caterpillars on S. We denote the reconfiguration graphs on $\mathscr{C}(S)$ by $G^{\text{flip}}_{\mathscr{C}}(S)$, $G^{\text{comp-flip}}_{\mathscr{C}}(S)$, $G^{\text{rot}}_{\mathscr{C}}(S)$, $G^{\text{emp-rot}}_{\mathscr{C}}(S)$, $G^{\text{slide}}_{\mathscr{C}}(S)$. In this direction, we prove the following:

Theorem 1. *Let S be a set of $n \geq 3$ points in convex position in the plane. Then, the graph $G^{slide}_{\mathscr{C}}(S)$ is connected with diameter at most $3n-8$.*

Theorem 2. *Let S be a set of n points in general position. If $n \leq 7$, then $G^{slide}_{\mathscr{C}}(S)$ is always connected. If $n \geq 8$, there exists a set S_n of n points such that $G^{slide}_{\mathscr{C}}(S)$ has isolated vertices. For all sets S of size n, $G^{slide}_{\mathscr{C}}(S)$ contains a connected component of size $\frac{1}{4}(3^n - 1)$.*

Theorem 3. *Let S be a set of n points in general position, then the graph $G^{rot}_{\mathscr{C}}(S)$ is connected.*

This is joint work with Todor Antić and Guillermo Gamboa Quintero.

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Separating subsets from their images

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Let G be a transitive permutation group of degree n. Let $\mathbf{m}(G)$ be the largest integer such that, for every set A of this size, we are guaranteed the existence of a permutation $g \in G$ such that $A \cap A^g$ is empty. By Neumann's Separation Lemma, we know that $\mathbf{m}(G) \geq \sqrt{n}$. Experimental evidence suggests that, unless G contains a large alternating subgroup, $\mathbf{m}(G)$ grows asymptotically as $\mathscr{O}(\sqrt{n})$. We discuss for which families of permutation groups we can currently establish this expected bound (*e.g.*, automorphism groups of k-valent graphs whose vertex-stabiliser has bounded order).

This is joint work with Marco Barbieri, Kamilla Rekvényi and Primož Potočnik.

The diameter of difference graphs of nilpotent groups

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Given a group G, the *power graph* of G is the simple graph whose vertex set is G, such that two vertices are adjacent if one is a power of the other. The *enhanced power graph* of G

is the simple graph with vertex set G, where two vertices are adjacent if they are contained in the same cyclic subgroup.

The concept of the difference graph was first introduced by Biswas *et al.* Given a finite group G, the *difference graph* of G is defined as the difference between the enhanced power graph and the power graph of G, with all isolated vertices removed. Although power and enhanced power graphs have been extensively studied, the structure of the difference graph remains less understood.

In this talk, we focus on nilpotent groups and study the graph-theoretic properties of their difference graphs, especially the diameter and domination number. In particular, we prove that the diameter of the difference graph of a nilpotent group has an upper bound of 4. Furthermore, we generalize and refine the result previously obtained by Biswas *et al.* by classifying all nilpotent groups whose difference graph has diameter k, for each $k \leq 4$. This classification is closely related to the structure of the Sylow p-subgroups and the exponent of the group.

This is joint work with Xuanlong Ma and Samir Zahirović.

Nut graphs with two vertex and three edge orbits

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A nut graph is a graph whose adjacency matrix is singular with one-dimensional null space spanned by a vector with no zero entries. In [1], the authors proved that the automorphism group of a nut graph has more orbits on the edge set than on the vertex set. They classified all orders for which a vertex-transitive nut graph with precisely two edge orbits exists, and conjectured that a nut graph with two vertex and three edge orbits exists for each non-prime order $n \geq 9$. Moreover, in [1], this conjecture has been confirmed for all even n, but it remains largely unsolved for odd n.

In this talk, we present some properties of nut graphs of odd order admitting a subgroup of automorphisms with two vertex and three edge orbits. We then introduce a general construction of graphs admitting a subgroup of automorphisms with two vertex and three edge orbits, and discuss some sufficient spectral and structural conditions under which these graphs are nut graphs and have precisely two vertex and three edge orbits. We also provide several infinite families of nut graphs with two vertex and three edge orbits arising from this construction, and illustrate some examples. Members of these infinite families confirm the above conjecture for all odd non-prime orders up to 2 500 and for at least 99.8 percent of all odd non-prime orders up to one million. Finally, some noteworthy examples of nut graphs with two vertex and three edge orbits that do not arise from the aforementioned construction are presented.

This is joint work with Primož Šparl.

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Doubly even self-orthogonal codes from quasi-symmetric designs

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In this talk, we give some constructions of doubly even self-orthogonal codes from incidence and orbit matrices of quasi-symmetric designs. Quasi-symmetric designs are combinatorial designs having the property that there are exactly two different block intersection numbers. In this talk, we will also be interested in specific quasi-symmetric designs of Blokhuis-Haemers type, which we will use in some of our constructions. Finally, as an illustration of the method, we will obtain doubly even and self-orthogonal codes that are optimal.

This is joint work with Dean Crnković, Doris Dumičić Danilović, Andrea Švob. This talk is based on the following paper.

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Partial automorphisms and Asymmetric depth of graphs

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While it is well-established that almost all graphs are asymmetric, possessing no non-trivial global automorphisms, all graphs contain non-trivial local symmetries which we study using isomorphisms between induced subgraphs, known as partial automorphisms. The set of all partial automorphisms, along with the operations of partial composition and partial inverse of partial maps, forms an inverse monoid, which is a rich and complex algebraic structure. However, it is hard to compute. In this talk, we are motivated by the study of partial automorphism inverse monoids of graphs initiated by [1]. We investigate the extent of these local symmetries through the measure of asymmetric depth of graphs defined through the rank of the largest non-trivial partial automorphism. In [2], we established a new, tight lower bound for the asymmetric depth of any simple graph Γ on n vertices. Any graph achieving this bound must be a strongly regular graph with parameters $(n, \frac{n-1}{2}, \frac{n-5}{4}, \frac{n-1}{4})$ also known as *conference graph*. We implemented a parallel algorithm for checking asymmetric depth on a high-performance cluster. Using this algorithm, we identified an asymmetric conference graph on 37 vertices that meets this bound, thereby proving its tightness. We also showed that it is one of the smallest possible graphs to meet this bound by checking all asymmetric conference graphs up to 37 vertices.

This is joint work with Tatiana Jajcayová.

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STUDENT TALKS – THURSDAY, 11 SEPTEMBER

Unavoidable subgraphs in digraphs with large out-degrees

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We ask the question, which oriented trees T must be contained as subgraphs in every finite directed graph of sufficiently large minimum out-degree. We formulate the following simple condition: all vertices in T of in-degree at least 2 must be on the same 'level' in the natural height function of T. We prove this condition to be necessary and conjecture it to be sufficient. In support of our conjecture, we prove it for a fairly general class of trees.

An essential tool in the latter proof, and a question interesting in its own right, is finding large subdivided in-stars in a directed graph of large minimum out-degree. We conjecture that any digraph and oriented graph of minimum out-degree at least $k\ell$ and $k\ell/2$, respectively, contains the $(k-1)$-subdivision of the in-star with ℓ leaves as a subgraph; this would be tight and generalizes a conjecture of Thomassé. We prove this for digraphs and $k=2$ up to a factor of less than 4.

This is joint work with Tomáš Hons, Tereza Klimošová, David Mikšaník, Mykhaylo Tyomkyn and Josef Tkadlec.

The limits of enumeration methods

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The methods with which we enummerate and count things can vary significantly based on the problem at hand. In this talk, we focus on applying various approaches to the following problem, which has already been explored in [1, 2]:

Definition 1. *Let $\mathscr{A}$ be a fixed alphabet of symbols and for fixed $k, l \in \mathbb{Z}^+$, let Z be a set of $k \times l$ matrices over $\mathscr{A}$. Then a **boundary matrix** (w.r.t. $\mathscr{A}, Z$) is any matrix over alphabet $\mathscr{A}$ which does not contain any matrix $Z_i \in Z$ as a contiguous submatrix.*

The problem we focus on is determining, for fixed size $m \times n$, the number of boundary matrices as a function $B(\mathscr{A}, Z, m, n)$. It is known that

Theorem 1. *For $m \in \mathbb{Z}^+$ fixed, there exists a linear recurrence relation in n for the function values of $B(\mathscr{A}, Z, m, n)$.*

However, it is not known if there exists a "two dimensional linear recurrence relation" for the values of B, which would be a much more powerful way of computing these values. This is one of our main focus points in this research project and as we will see, such a relation often does not exist.

Remark. Boundary matrices are an example of structures that are locally testable; a matrix is boundary, if non of its windows have the forbidden pattern(s). Related ideas have been explored in the context of block gluing [3] and in measuring the entropy of ice [4].

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Generating optimal mixed strategies for `Shortest Path with Adversary` combinatorial game

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For many combinatorial games, finding optimal strategies for players is a challenging problem. In many cases, we want to find the strategies for a specific game instance (e.g., for the fixed graph). During the talk, we will present the procedure based on Mixed Integer Linear Programming to efficiently generate optimal mixed strategies for min-max `Shortest Path with Adversary` game.

The game takes place on a weighted graph G. One player (called *Pather*) chooses the path in the graph G between two fixed vertices. The another player (called *Attacker*) has the fixed budget that he can distribute among the edges of the graph, thereby increasing the weights of some edges. By mixed strategy we mean that the player has a probability distribution over the space of player's moves.

This is joint work with Armin Fügenschuh and Konstanty Junosza-Szaniawski.

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Sequence variant of irregularity strength for hypercubes

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In the paper [2] the authors write about sequence variant of irregularity strength. This talk is about distinguishing vertices of a hypercube by sequences [1]. In a hypercube, we can define the order of edges, which results from the structure of this graph. We start by coloring the edges of an n - dimensional hypercube. Next, we can assign a sequence of colors to each vertex in such a way that the i-th element of this sequence is the color of the i-th edge coming from this vertex. We want to find a minimum number of colors to distinguish each pair of vertices in an n-dimensional hypercube.

This is joint work with Aleksandra Gorzkowska and Mariusz Woźniak.

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Orientations, decompositions and rigidity of highly connected graphs

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Let G be a graph whose vertices are placed in $\mathbb{R}^d$, and let us think of the edges of G as straight bars. Such a *framework* is called *rigid* if there is no continuous motion of the vertices that preserves all edge lengths but changes the distance between at least one pair of vertices. For 'sufficiently general' placements of the vertices, rigidity depends only on the underlying graph G and the dimension d. We therefore say G is *rigid in* $\mathbb{R}^d$ if its generic frameworks are rigid. Lovász and Yemini proved in 1982 that every 6-connected graph is rigid in $\mathbb{R}^2$, and conjectured that for every d there is a constant $f(d)$ such that all $f(d)$-connected graphs are rigid in $\mathbb{R}^d$. This conjecture was recently resolved:

Theorem 1. [2] *Every $d(d+1)$-connected graph is rigid in $\mathbb{R}^d$.*

This talk will briefly outline the proof of Theorem 1, which combines matroid theory with the probabilistic method. We then discuss how rigidity theory and this proof method were applied in [1] to prove the following conjectures on graph connectivity:

For every integer k, there exists an integer $f(k)$ such that:

Conjecture 1. (Kriesell, 2003) *Every $f(k)$-connected graph has an edge-disjoint k-connected spanning subgraph and a spanning tree.*

Conjecture 2. (Garamvölgyi, Jordán, Király, 2023) *Every $(t \cdot f(k))$-connected graph has t edge-disjoint k-connected spanning subgraphs.*

Conjecture 3. (Thomassen, 1985) *Every $f(k)$-connected graph has a k-connected orientation.*

This is joint work with Dániel Garamvölgyi, Tibor Jordán and Csaba Király.

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Pass the buck and resistance distance

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In this talk, we present a simple game called Pass the Buck and show how it can be used to derive resistance distances in certain types of graphs. We focus in particular on the Wheel and Fan graphs. While several recent papers (e.g., [1, 2, 3, 4]) have provided explicit formulas for resistance distance in various graph classes, our approach builds on the results of [1, 5], offering an alternative derivation using the dynamics of the Pass the Buck game. This method can be extended to broader families of graphs.

This is joint work with Antonín Slavík.

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α-labeling of banana tree graphs

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A graceful labeling assigns the numbers 0 through $|E|$ to vertices so that edge differences yield all values from 1 to $|E|$ exactly once; an α-labeling is a graceful labeling with an additional ordering property, motivated by work of Rosa [1], and is central in graph decomposition theory.

A banana tree graph, denoted by $B(n,k)$, is a tree constructed by connecting n star graphs S_k to a root vertex. This result builds upon previous research on graceful labelings of banana trees and explores their α-labelability. By leveraging the symmetries of B(n,k), we enumerate all non-isomorphic α-labelings through the definitions of *proper* and *central* α-labelings. We achieve a complete classification of all α-labelings for all the $B(n,1)$, $B(1,k)$ and $B(2,k)$ trees. We present α-labelings of the $B(n,k)$ tree, for $k \geq \frac{n}{2}+1$ and $n \in \mathbb{N}$. Finally, we prove that the $B(n,k)$ tree is not α-labelable for $1 < k \leq \frac{n}{2}$ and any $n \geq 3$, classifying all the $B(n,k)$ trees based on their α-labelability.

This is joint work with Anamari Nakić.

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Additive graph decompositions

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A Γ-*decomposition* of a simple graph K is a family of subgraphs of K whose edge sets partition the edge set of K, and such that each subgraph in the family is isomorphic to a fixed graph Γ. We say that a graph decomposition of K is *additive* if we can label the vertices of K with elements of an abelian group in such a way that each subgraph in the decomposition is zero-sum.

This definition was first introduced in [1] as a generalization of additive block designs, which were first studied in [2], motivated by the frequent use of zero-sum blocks in design constructions. Particular emphasis was given to additive Γ-decompositions of the complete graph K_v, referred to as *additive* (K_v,Γ)*-designs.*

We will discuss difference methods used for graph decomposition, as well as a *coseted strategy* for establishing the additivity of a (K_v,Γ)-design.

This is joint work with Anamari Nakić.

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Approximate EFX allocations in Hypergraphs

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In this ongoing work, we study the problem of allocating a set of indivisible goods to a set of agents with subadditive valuation functions. Our goal is to achieve approximate envy-freeness up to any good ($\alpha - EFX$) in the hypergraph setting. We are restricted in hypergraphs of girth 3 and in the class of hypergraphs of multiplicity 2, where the vertices correspond to agents and the hyperedges to items.

This is joint work with Alkmini Sgouritsa.

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Exploring supertoken graphs

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In graph theory, it is common to construct “larger" graphs from “smaller" ones, leading to a fundamental question: how can the properties of the resulting graph be inferred or approximated based on the characteristics of the original? Among such constructions, token graphs, introduced by Fabila-Monroy, Flores-Peñaloza, Huemer, Hurtado, Urrutia, and Wood in [2] have garnered significant attention. In this talk, we focus on a particular variation called supertoken graphs, which belong to a broader family of structures we term generalized token graphs.

In the k-super token graph, each vertex corresponds to a configuration of k tokens (distinguished or undistinguished) distributed over the vertices of a base graph G. An edge connects two vertices (or configurations) when one configuration can be transformed into the other by moving one or more tokens along the edges of G. Hammack and Smith first explored this concept in [3] under the name reduced k-th power of graphs, later referred to as k-supertokens graph by Baskoro, Dalfó, Fiol, and Simanjuntak in [1].

In this presentation, we deal with some properties of supertoken graphs and highlight their connections to other well-known graph constructions.

This is joint work with Cristina Dalfó and Miquel Àngel Fiol.

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Eccentric graph of trees and their Cartesian products

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Let G be an undirected simple connected graph. We say a vertex u is eccentric to a vertex v in G if $d(u,v) = \max\{d(v,w) : w \in V(G)\}$. The eccentric graph of G, say $Ec(G)$, is a graph defined on the same vertex set as of G and two vertices are adjacent if one is eccentric to the other. We find the structure and the girth of the eccentric graph of trees and see that the girth of the eccentric graph of a tree can either be zero, three, or four. Further, we study the structure of the eccentric graph of the Cartesian product of graphs and prove that the girth of the eccentric graph of the Cartesian product of trees can only be zero, three, four or six. Furthermore, we provide a comprehensive classification when the eccentric girth assumes these values. We also give the structure of the eccentric graph of the grid graphs and the Cartesian product of two cycles. Finally, we determine the conditions under which the eccentricity matrix of the Cartesian product of trees becomes invertible.

This is joint work with Anita Arora.

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A FEW WORDS ABOUT SLOVENIAN DISCRETE AND APPLIED MATHEMATICS SOCIETY

Slovenian Discrete and Applied Mathematics Society was founded in Koper (Slovenia), on 14 December 2016. The aim of this society is to promote the mathematical sciences, with special emphasis given to discrete and applied mathematics. The Society is research-oriented, and publishes scientific literature and organises scientific meetings such as this one. In particular, it is involved in publishing *Ars Mathematica Contemporanea* and *The Art of Discrete and Applied Mathematics* . It has members, fellows and honorary members.

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- AC^2 – Algebraic Combinatorics on the Adriatic Coast, Koper, 2003, 2004, 2008, 2009.
- International Workshop on Symmetries of Graphs and Networks 2010, 2012, 2014.
- PhD Summer Schools in Algebraic Graph Theory 2011 and Discrete Mathematics 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2022, 2023.
- 7th Slovenian International Conference on Graph Theory, Bled, 2011.
- Computers in Scientific Discovery 6, August 2012.
- $DM = 60$ Conference on Graph Theory and Combinatorics, May 2013.
- Joint Conference of Catalan, Slovak, Austrian, Slovenian and Czech Math. Society, June 2013.
- International Conference on Graph Theory and Combinatorics, May 2014.
- Ljubljana - Leoben Graph Theory Seminar 2014, September 2014.
- 2015 International Conference on Graph Theory, May 2015.
- Algorithmic Graph Theory on the Adriatic Coast, June 2015.
- 8^{th} Slovenian International Conference on Graph Theory, Kranjska Gora, June, 2015.
- Graphs, groups, and more: celebrating Brian Alspach's 80th and Dragan Marušič's 65th birthdays, Koper, May-June, 2018.
- 8ECM: European Congress of Mathematics, Portorož, June, 2021.
- The 10th Workshop on Graph Classes, Optimization, and Width Parameters - GROW 2022, Koper, September, 2022.
- Groups, graphs and everything in between, Bled, May, 2023.
- 50th International Workshop on Graph-Theoretic Concepts in Computer Science, Kranjska Gora, June, 2024.
- Combinatorics around the q-Onsager algebra, Kranjska Gora, June 2025.

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ARS MATHEMATICA CONTEMPORANEA

Ars Mathematica Contemporanea (AMC) is an international journal, published by UP in collaboration with IMFM, the Slovenian Discrete and Applied Mathematics Society and the Slovenian Society of Mathematicians, Physicists and Astronomers.

The aim of AMC is to publish peer-reviewed high-quality articles in contemporary mathematics that arise from the discrete and concrete mathematics paradigm. It favors themes that combine at least two different fields of mathematics. In particular, papers intersecting discrete mathematics with other branches of mathematics, such as algebra, geometry, topology, theoretical computer science, and combinatorics, are most welcome.

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This journal was launched in 2008 by Tomaž Pisanski and Dragan Marušič, and the editorial board today is led by Editors in Chief, Klavdija Kutnar, Dragan Marušič and Tomaž Pisanski.

For more information on submissions, please refer to the AMC website

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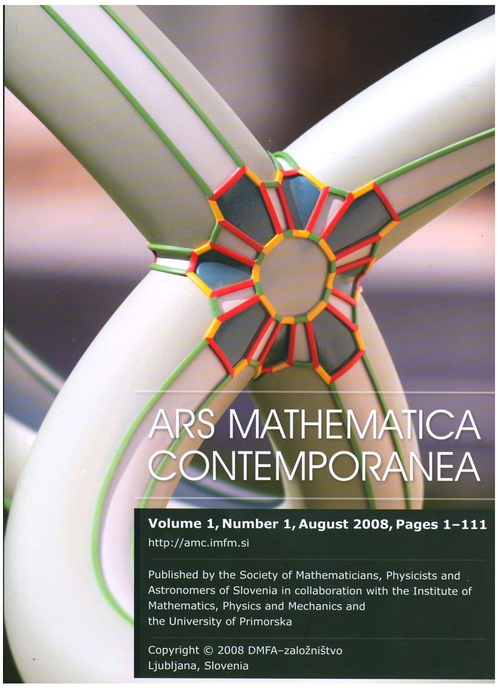

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