Book of Abstracts

Combinatorics around the q-Onsager algebra

A CONFERENCE CELEBRATING THE WORK OF

Paul Terwilliger

WELCOME

Dear conference participants,

Welcome to Kranjska Gora, Slovenia, for the **Combinatorics around the q-Onsager Algebra Conference**, affectionately known as TerwilligerFEST. We gather here to honor Paul Terwilliger on the occasion of his 70th birthday and to celebrate his significant contributions to mathematics.

Paul's remarkable work spans a wide range of topics, including distance-regular graphs, association schemes, Leonard pairs, tridiagonal pairs, orthogonal polynomials from the Askey scheme, the q-Onsager algebra, and more. For decades, his research has profoundly shaped and deepened our understanding of these areas, and it has opened new avenues of research that continue to inspire mathematicians around the globe. At the same time, Paul has touched so many of our lives, both mathematical and personal, with his immense generosity and cheerful spirit.

At this time we would like to thank our sponsors for their financial support, including the University of Primorska, the Slovenian Discrete and Applied Mathematics Society, and the Institute of Mathematics, Physics and Mechanics. We also wish to thank the Journal of Algebraic Combinatorics for devoting a special issue to our conference proceedings. And we are most grateful for the hard work and dedication of the members of our organizing committee: Blas Fernández, Giusy Monzillo, and Safet Penjić.

We hope this conference will be a fitting tribute to Paul's remarkable career and a joyful celebration of his 70th birthday. Let us take this opportunity to engage in fruitful discussions, share fond memories, and express our deep gratitude for Paul's invaluable contributions to our community.

GENERAL INFORMATION

TerwilligerFest — Combinatorics around the q-Onsager Algebra

Ramada Resort, Kranjska Gora, Slovenia | June 22/23–27, 2025

About the Conference:

We will celebrate the 70th birthday of Paul Terwilliger with a conference dedicated to the wide range of mathematical topics he has contributed to throughout his career, all of which are connected to the q-Onsager algebra. These topics include:

- **Algebraic graph theory:** topics include distance-regular graphs, the subconstituent algebra, association schemes, and the Q-polynomial property.
- **Linear algebra:** Leonard pairs, tridiagonal pairs, billiard arrays, lowering-raising triples, and a linear algebraic approach to the orthogonal polynomials of the Askey scheme;
- **Lie theory:** the tetrahedron algebra and the Onsager algebra;
- **Algebras and their representations:** topics include the equitable presentation of $U_q(\mathfrak{sl}_2)$, the q-Onsager algebra arising in mathematical physics, the q-tetrahedron algebra, and the universal Askey–Wilson algebra.

The conference program includes a welcoming reception and registration in the early evening of June 22. Talks will begin on the morning of June 23 and continue until the evening of Friday, June 27.

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Sponsored by

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IMFM — Institute of Mathematics, Physics and Mechanics;
SDAMS — Slovenian Discrete and Applied Mathematics Society.

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INVITED TALKS

Group algebras and quantum Latin squares

Arnbjörg Soffía Árnadóttir*, David Roberson

In this talk I will introduce quantum Latin squares (QLS) and their connection to quantum isomorphisms of graphs. I will define the notion of invariance of a QLS under a pair of groups, (G,G') and give a simple condition on G and G' for such a QLS to exist (spoiler: it has to do with their algebras). Finally I will show how this can in theory help us construct pairs of quantum isomorphic Cayley graphs for G and G', respectively. The talk is based on the following preprint: https://arxiv.org/abs/2501.00196. No quantum knowledge is required.

Compact symmetric spaces (Compact Gelfand pairs) vs. finite Gelfand pairs

Eiichi Bannai*

This talk is based on the ongoing joint work with Hirotake Kurihara (Yamaguchi University) and Da Zhao (East China University of Science and Technology).

Finite Gelfand pairs are finite multiplicity-free permutation groups and are special cases of commutative association schemes We want to study finite commutative association schemes (and finite Gelfand pairs) from the viewpoint of multivariate P-and Q-polynomial association schemes. We first remark our new observation that compact symmetric spaces (of arbitrary rank ℓ) can be regarded to have a kind of multivariate (ℓ-variate) P-and Q-polynomial property in our sense. Then we discuss which finite Gelfand pairs (or something close to Gelfand pairs) can be regarded as corresponding to each of the compact symmetric spaces that were classified classically by E. Cartan.

A correspondence between Leonard pairs, orthogonal polynomials and the algebraic Bethe ansatz

Pascal Baseilhac*

Introduced and developed by Paul Terwilliger and collaborators, the theory of Leonard pairs gives the (finite dimensional) representation theoretic framework for the Askey-Wilson algebra and discrete orthogonal polynomials of the Askey-scheme. Introduced and developed by St Pertersburg's school starting from the 80's, the Yang-Baxter, reflection algebras and algebraic Bethe ansatz have played a central role for solving quantum integrable systems such as spin chains with periodic or generic boundary conditions. In this talk, I will present a precise correspondence between both framework: eigenbases for Leonard pairs and triples versus Bethe states of homogeneous and inhomogeneous type; q-Racah polynomials as scalar products of Bethe states; Zeroes of Askey-Wilson polynomials and solutions of Bethe equations of inhomogenous type. This correspondence allows to establish the existence of determinant representations for q-Racah polynomials, as solutions of Belliard-Slavnov's linear systems derived in the

context of the algebraic Bethe ansatz. The extension of the correspondence to the theory of tridiagonal pairs that gives the (finite dimensional) representation theoretic framework for the q-Onsager algebra will be briefly sketched. Based on joint works with R.P. Pimenta arXiv:2211.14727; arXiv:2501.10310.

An Algebra Associated to Isotropic Subspaces

Pierre-Antoine Bernard*, Luc Vinet, Meri Zaimi, Xiaohong Zhang, Nicolas Crampé

This talk presents ongoing work on an algebraic framework connected to the bispectral properties of bivariate orthogonal polynomials arising from the study of isotropic subspaces.

The Askey-Wilson algebra encapsulates the bispectrality of orthogonal polynomials in the Askey scheme and plays a central role in the theory of Leonard pairs as well as in the structure of P- and Q-polynomial association schemes. Recently, multivariate extensions of these schemes have been proposed. As in the univariate case, the corresponding subconstituent algebras are expected to relate to higher-rank generalizations of the Askey-Wilson algebra. However, results in this direction remain sparse.

This talk will present preliminary results on association schemes constructed from isotropic subspaces and will explore their connection with the algebraic structure of uniform posets.

Metathin table algebras and association schemes

Harvey Blau*, Bangteng Xu, Caroline Wroblewsk

Metathin standard table algebras (table algebra extensions of one group by another) are characterized in terms of factor sets and other machinery, in a manner analogous to M. Suzuki's description of group extensions and Bang and Hirasaka's construction of metathin association schemes. Necessary and sufficient criteria are derived, in terms of a relevant factor set, for when a metathin table algebra occurs as the adjacency algebra of an association scheme; and, in terms of the two relevant factor sets, for when two metathin association schemes, whose adjacency algebras appear as two isomorphic table algebras, are in fact combinatorially isomorphic.

Tridiagonal pairs of Racah type, the double lowering operator Ψ, and the universal enveloping algebra $U(\mathfrak{sl}_2)$

Sarah Bockting-Conrad*

Let $\mathbb{F}$ denote a field and let V denote a vector space over $\mathbb{F}$ with finite positive dimension. We consider an ordered pair of linear transformations $A : V \to V$ and $A^* : V \to V$ that satisfy the following four conditions: (i) Each of A, A^* is diagonalizable; (ii) there exists an ordering $\{V_i\}_{i=0}^d$ of the eigenspaces of A such that $A^*V_i \subseteq V_{i-1} + V_i + V_{i+1}$ for $0 \leq i \leq d$, where $V_{-1} = 0$ and $V_{d+1} = 0$; (iii) there exists an ordering $\{V_i^*\}_{i=0}^\delta$ of the eigenspaces

of A^* such that $AV_i^* \subseteq V_{i-1}^* + V_i^* + V_{i+1}^*$ for $0 \leq i \leq \delta$, where $V_{-1}^* = 0$ and $V_{\delta+1}^* = 0$; (iv) there does not exist a subspace W of V such that $AW \subseteq W$, $A^*W \subseteq W$, $W \neq 0$, $W \neq V$. We call such a pair a tridiagonal pair on V.

In this talk, we will consider a special class of tridiagonal pairs said to have Racah type. Given a tridiagonal pair of Racah type, we associate with it several linear transformations which act on the underlying vector space in an attractive manner and discuss their relationships with one another. In an earlier work, we introduced the double lowering operator Ψ for a tridiagonal pair. In this talk, we will explore this double lowering map further under the assumption that our tridiagonal pair has Racah type and will use the double lowering map to obtain new relations involving the operators associated with two oriented versions of our tridiagonal pair.

Doubly Almost Bipartite Leonard Pairs

John Caughman*, Shuichi Masuda

Let $\mathbb{K}$ denote a field and let d be a positive integer. Let V be a vector space of dimension $d+1$ over $\mathbb{K}$. A Leonard pair on V is an ordered pair (A, A^*) of diagonalizable linear maps on V, with the property that each acts on an eigenbasis for the other one in an irreducible tridiagonal fashion. In this talk, we define what it means for (A, A^*) to be *doubly almost-bipartite*. In our main result, we classify (up to isomorphism) the doubly almost-bipartite Leonard pairs.

Complex Hadamard matrices and quantum symmetry

Ada Chan*

An $n \times n$ matrix W is type-II if $W W^{(-)T} = nI$, where $W^{(-)}$ denotes the Schur-inverse of W. A complex Hadamard matrix is a type-II matrix whose entries have absolute value one.

In this talk, we construct a quantum permutation matrix from a complex Hadamard matrix W, and give sufficient conditions for which the graphs in the Nomura algebra of W^T have quantum symmetry.

This is joint work with Chris Godsil and Thomás Jung Spier.

EKR sets of flags in finite spherical buildings

Jan de Beule*, Philipp Heering, Sam Mattheus, Klaus Metsch

Recall the following theorem, by Erdős, Ko, and Rado (EKR), [1].

Let $n \in \mathbb{N}$. Let $\mathscr{F}$ be a family of pairwise intersecting k-subsets of $\{1, \ldots, n\}$. If $n \geq 2k$, then $|\mathscr{F}| \leq \binom{n-1}{k-1}$, and, furthermore, in case of equality and $n > 2k$, $\mathscr{F}$ is the set of all k-subsets that contain a given element.

The set $\mathscr{F}$ is a co-clique in the Kneser graph $K_{n;k}$. Hoffman's ratio bound gives immediately an upper bound on the size of $\mathscr{F}$, while the characterization in case of equality

follows from the fact that the characteristic vector of a co-clique is orthogonal to one of the eigenspaces of the adjancency matrix of the graph $K_{n;k}$. This is one of the known proofs for the theorem, and it has been inspiring to investigate many EKR-type probles in *finite geometry*.

In this talk, we will first give some typical examples of EKR theorems in finite geometry. Many of them are proved by representing the problem as a co-clique problem in a graph, or more generally, as a co-clique problem in an association scheme. We discuss how an algebraic approach leads to EKR-type theorems.

We then discuss an EKR type problem on flags of a finite geometry. We explain the motivation to study this type of problem and then carefully explain how this type of problem is represented by a problem in a non-commutative association scheme.

BIBLIOGRAPHY

[1] P. Erdős, C. Ko, and R. Rado. Intersection theorems for systems of finite sets. *Quart. J. Math. Oxford Ser. (2)*, 12:313–320, 1961.

Extending the 2-homogeneous property to distance-biregular graphs

Blas Fernández*

The concept of *2-homogeneity*, introduced by Kazumasa Nomura in 1994, fully characterized within the setting of bipartite distance-regular graphs, provides a rich combinatorial structure often approached via algebraic methods. In this work, we propose a natural extension of this notion, termed the *2-Y-homogeneous* property, defined for bipartite graphs with vertex bipartition (Y, Y'), where all vertices in Y have the same eccentricity. While this generalization aligns with classical 2-homogeneity in the distance-regular context, it reveals more intricate behavior beyond that framework.

We investigate this phenomenon in the context of *distance-biregular graphs* (DBRGs), a class of graphs closely related to distance-regular graphs. Our focus is on characterizing DBRGs that satisfy the 2-Y-homogeneous condition by identifying structural constraints that inform their classification. These graphs serve as a nexus between algebraic graph theory, combinatorial design theory, and the representation theory of Terwilliger algebras.

This talk will present recent developments in the structural analysis and classification of 2-Y-homogeneous DBRGs—including new characterizations, illustrative examples, and open problems that point toward promising directions for future research.

Twin buildings and Hypergroups, II

Christopher French*, Paul-Hermann Zieschang

This talk will continue Zieschang's talk on twin buildings and hypergroups. In particular, we will consider the building theoretic condition that would imply an identification of twin buildings with actions of twin Coxeter hypergroups. In a building (X, W), one has a set X of elements, called chambers, and one can measure a kind of

'distance' between each pair of chambers with an element of a Coxeter group W. That is, each element $v \in W$ determines a subset f_v of $X \times X$. In a twin building, one has a pair of buildings (X_+, W) and (X_-, W) (with the same Coxeter group), and one can measure a 'codistance' between two chambers, with one chamber coming from X_- and the other from X_+. These codistances are also elements of W, so again, each element t of the Coxeter group W determines a subset r_t on $(X_+ \times X_-) \cup (X_- \times X_+)$. The building theoretic condition we require is that for any three elements $t, u, v \in W$, if $f_v \cap (r_t \circ r_u)$ is nonempty, then $f_v \subseteq r_t \circ r_u$. We will discuss progress toward a proof that the class of "thick" twin buildings satisfy this property, considering in particular the cases where the element v has small Coxeter length.

The q-Onsager algebra and the quantum torus

Owen Goff*

The q-Onsager algebra, denoted O_q, is defined by two generators W_0, W_1 and two relations called the q-Dolan-Grady relations. The quantum torus, denoted T_q, is defined by generators x, y and their inverses and the relation $xy = q^2 yx$. The set $\{x^i y^j | i, j \in \mathbb{Z}\}$ is a basis for T_q. There exists an algebra homomorphism $p : O_q \mapsto T_q$ that sends $W_0 \mapsto x + x^{-1}$ and $W_1 \mapsto y + y^{-1}$.

In their 2020 paper, Baseilhac and Kolb introduced the Baseilhac-Kolb elements of O_q. In his 2022 paper, Terwilliger defined the alternating elements of O_q. In their 2021 paper, Lu and Wang effectively defined some elements of O_q that we call the Lu-Wang elements of O_q.

The aforementioned elements of O_q are defined using recursive formulas and generating functions, and it is difficult to express them in closed form. To mitigate this difficulty, we map everything to T_q using p. In this talk we show how to find the p-images of these elements, presenting them in the basis for T_q and also in attractive closed forms.

The coherent closure of a Neumaier graph

Gary Greaves*

The *coherent closure* of a graph is the smallest coherent algebra containing its adjacency matrix. A *Neumaier graph* is an edge-regular graph that contains a regular clique. Many known Neumaier graphs are strongly regular, and these can be identified by the fact that their coherent closures have rank 3. More generally, the coherent closure serves as a useful tool for distinguishing between different families of graphs. The first non-strongly-regular Neumaier graphs were discovered in 2018 by Greaves and Koolen and are known as *strictly Neumaier graphs*. In this work, we investigate Neumaier graphs whose coherent closures have small rank.

As a byproduct of this study, we present a new construction of Neumaier graphs from cyclotomy. A Neumaier graph Γ is said to have parameters $(v, k, \lambda; e, s)$ if it is (v, k, λ)-edge-regular and contains an e-regular clique of order s. Neumaier (1981) showed that the regular cliques of Γ are precisely its maximal cliques, all of which are e-regular of order s; the parameter e is called the *nexus* of Γ. Most known constructions of strictly Neumaier graphs have nexus $e = 1$. The first examples with $e > 1$ were constructed by

Evans, Goryainov, and Panasenko (2019), but in all of their families, the nexus is a power of 2. They asked whether strictly Neumaier graphs exist whose nexus is not a power of 2. Our new construction answers this question in the affirmative.

On the centrally primitive idempotents of Terwilliger algebras

Allen Herman*

In a recent paper, [AH, R. Maleki, and A. S. Razafimahatratra, On the Terwilliger algebra of the group association scheme of the symmetric group $Sym(7)$, J. Combin. Des., 2025], Mitra Maleki, Bidy Razafimahatratra and I determined the dimension and Wedderburn decomposition of the Terwilliger algebra of the conjugacy class scheme of the symmetric group S_7. To determine the Wedderburn decomposition, the centrally primitive idempotents of this Terwilliger algebra were deduced explicitly from those of a slightly larger centralizer algebra, whose centrally primitive idempotents have a natural character formula. In my talk I will discuss the nature of centrally primitive idempotents of Terwilliger algebras in general, and describe situations where our techniques can be helpful.

A skew group ring of $\mathbb{Z}/2\mathbb{Z}$ over $U(\mathfrak{sl}_2)$, Leonard triples and odd graphs

Hau-Wen Huang*, Chin-Yen Lee

The universal enveloping algebra $U(\mathfrak{sl}_2)$ of $\mathfrak{sl}_2$ is an algebra over $\mathbb{C}$ generated by E, F, H subject to the relations $[H,E]=2E$, $[H,F]=-2F$, $[E,F]=H$. Merging with the algebra automorphism ρ of $U(\mathfrak{sl}_2)$ given by $(E,F,H)\mapsto(F,E,-H)$, this produces a skew group ring $U(\mathfrak{sl}_2)_{\mathbb{Z}/2\mathbb{Z}}$ of $\mathbb{Z}/2\mathbb{Z}$ over $U(\mathfrak{sl}_2)$. The skew group ring $U(\mathfrak{sl}_2)_{\mathbb{Z}/2\mathbb{Z}}$ is a Hopf algebra. Let Δ denote the comultiplication of $U(\mathfrak{sl}_2)_{\mathbb{Z}/2\mathbb{Z}}$. The universal Bannai–Ito algebra $\mathfrak{BI}$ is an algebra over $\mathbb{C}$ with generators X, Y, Z and the relations assert that each of $\{X,Y\}-Z$, $\{Y,Z\}-X$, $\{Z,X\}-Y$ is central in $\mathfrak{BI}$. Let V denote a $U(\mathfrak{sl}_2)^{\otimes 2}_{\mathbb{Z}/2\mathbb{Z}}$-module. For any $\theta\in\mathbb{C}$ let $V(\theta)$ consist of all $v\in V$ with $\Delta(H)v=\theta v$. We show that $V(1)$ is a $\mathfrak{BI}$-module given by

$$\begin{aligned} X &= \Delta(E\rho),\\ Y &= \frac{H\otimes 1-1\otimes H}{2},\\ Z &= (E\otimes 1-1\otimes E)\Delta(\rho). \end{aligned}$$

Suppose that V is a finite-dimensional irreducible $U(\mathfrak{sl}_2)^{\otimes 2}_{\mathbb{Z}/2\mathbb{Z}}$-module. We show that X, Y, Z act on the $\mathfrak{BI}$-module $V(1)$ as a Leonard triple whenever $V(1)$ is nonzero. Let $d\geq 1$ be an integer. Fix a vertex x_0 of the odd graph O_{d+1}. Let $\mathbf{T}(x_0)$ denote the Terwilliger algebra of O_{d+1} with respect to x_0. As an application, we obtain an algebra homomorphism $\mathfrak{BI}\to\mathbf{T}(x_0)$ and X, Y, Z act on each irreducible $\mathbf{T}(x_0)$-module as a Leonard triple.

Almost commutative Terwilliger algebras of some Schur rings

Stephen Humphries*, Nicholas Bastian

We determine when the Terwilliger algebra $T(G)$ of the group association scheme $Z(\mathbb{C}[G])$ of the finite group G is almost commutative (meaning that there is exactly one irreducible $T(G)$-module of dimension greater than 1). Let $H \leq G$. We then generalize this situation to determine when the Schur ring $\mathbb{C}[G]^H$ of $\mathbb{C}[G]$ generated by the H-classes $g^H = \{g^h : h \in H\}$ is commutative. Here we need $\mathbb{C}[G]^H$ to be commutative, i.e. that (G,H) is a strong Gelfand pair. In each case we determine dim $T(G)$, the Wedderburn idempotents and then describe the association scheme (Schur ring) as a wreath product of some specific association schemes.

The Dunkl-Watanabe duality

Tatsuro Ito*

Let G be a general linear group $GL(N,D)$ over the finite field $\mathbb{F}_q$ and H a maximal parabolic subgroup of G. Then H acts naturally on the set X of all subspaces of $\mathbb{F}_q^N$. In an old paper published in 1977, C. Dunkl detemined the irreducible representations of H that appear in the permutation H-module $V = \mathbb{C}^X$.

In a relatively recent paper published in 2017, Y. Watanabe defined an algebra $\mathscr{H}$ that has a natural action on X, using combinatorial structures of the subspace lattice associated with X. He constructed an algebra homomorphism from the quantum affine algebra $U_q(\widehat{\mathfrak{sl}}_2)$ to $\mathscr{H}$, $\mathscr{H}$ being generated by the image and the centre of $\mathscr{H}$. He determined the irreducible representations of H that appear in the $\mathscr{H}$-module X.

We show that $\mathscr{H}$ coincides with the centralizer algebra of H acting on V, that is, $\mathscr{H} = \mathrm{End}_H(V)$. We call this relation the Dunkl-Watanabe duality.

The Terwilliger algebra of the q-Johnson scheme $J_q(N,D)$ is contained in the centralizer algebra of H acting on $J_q(N,D)$ and the question of whether they coincide or not remains open. By carefully examining the Dunkl-Watanabe duality, we show that they do not coincide. This result was also obtained recently by Hau-Wen Huang by a different approach.

This is joint work with Xiaoye Liang.

A new family of antipodal distance-regular graphs of diameter 3

Aleksandar Jurišić*

Antipodal distance-regular graphs of diameter 3 are studied. They are antipodal r-covers of a complete graph K_n for some $r, n \in \mathbb{N}$. All its intersection numbers are determined by the triple (n, r, c_2), where c_2 is the number of common neighbours of any two vertices at distance 2. The smallest n for which there are at least two covers is $n = 9$. In this case, we have $n-1 = (c_2+1)(r-1)$ and one can apply Brouwer's characterization, which implies that in certain cases the covers are the collinearity graphs of a generalized quadrangle GQ(s,t), where $s = r-1$ and $t = c_2+1$, with a spread (that is a set of lines that partition all the points) deleted. Our construction relies on regular spreads and produces a spread that is not regular. This is a joint work with Chris Godsil.

An improved bound for strongly regular graphs

Jack Koolen*

Sims showed that primitive strongly regular graphs with fixed smallest eigenvalue $-m$, except for a finite number of them, belong to two families of graphs. Later in 1979 Neumaier made this bound explicit. In this talk we will improve this bound by Neumaier. This is based on joint work with Chenhui Lv, Greg Markowsky and Jongyook Park.

On the b-chromatic number of rooted product graphs

Michael Lang*

The b-chromatic number of a graph G was defined by Irving and Manlove in 1999 as the largest integer k for which G admits a proper coloring with k colors such that every color class (in this proper coloring) has a vertex that is adjacent to at least one vertex in every other color class. The b-chromatic number has been studied in many contexts, including for various graph products. The rooted product, defined by Godsil and McKay in 1978, is not yet among these. We find bounds for the b-chromatic number of the rooted product of two graphs in terms of the b-chromatic numbers and degrees of the factors, along with some new parameters that we define. Moreover, we give sufficient conditions for equality to hold in these bounds. We refine our results, sometimes to exact values, when one or both of the factors is a path, cycle, complete graph, star, or wheel. This is joint work with Marko Jakovac and Sarah Bockting-Conrad.

Bipartite Coherent Configurations

Sabrina Lato*

Distance-biregular graphs are a class of bipartite graphs with strong algebraic and combinatorial strucutre, similar to distance-regular graphs. The same way that the adjacency algebra of distance-regular graphs can be described by association schemes, the adjacency algebra of distance-biregular graphs can be described by a specific class of coherent configurations, called *bipartite coherent configurations.* In this talk we will explore the properties of bipartite coherent configurations, especially as they parallel association schemes.

Four bases for the Onsager Lie algebra related by a $\mathbb{Z}_2 \times \mathbb{Z}_2$ action

Jae-Ho Lee*

The Onsager Lie algebra O is an infinite-dimensional Lie algebra defined by generators A, B and relations $[A,[A,[A,B]]] = 4[A,B]$ and $[B,[B,[B,A]]] = 4[B,A]$. Using an embedding of O into the tetrahedron Lie algebra, we obtain four direct sum decompositions of the vector space O, each consisting of three summands. In this talk, we show that there is a natural action of $\mathbb{Z}_2 \times \mathbb{Z}_2$ on these decompositions. For each decomposition, we provide

a basis for each summand. Moreover, we describe the Lie bracket action on these bases and show how they are recursively constructed from the generators A, B of O. Finally, we discuss the action of $\mathbb{Z}_2 \times \mathbb{Z}_2$ on these bases and determine some transition matrices among the bases.

On thin Jordan schemes

Misha Muzychuk*, Cristian Pech, Andrew Woldar

Let X be finite set, $M_X(\mathbb{C})$ the algebra of complex $X \times X$ matrices, $\cdot$ denotes the usual matrix product and $A \star B = \frac{1}{2}(AB + BA)$ stands for the Jordan product of matrices.

An *association scheme* (AS in brief) on a finite set X may be defined as a set of $0,1$-matrices $A_0, ..., A_d \in M_X(\mathbb{C})$ satisfying the following conditions

(A1) $A_0 = I_X$;

(A2) $\sum_{i=0}^{d} A_i = J_X$;

(A3) $\{A_0^t, A_1^t, ..., A_d^t\} = \{A_0, A_1, ..., A_d\}$;

(A4) The linear span $\mathscr{A} = \langle A_0, A_1, ..., A_d \rangle$ is subalgebra of $M_X(\mathbb{C})$.

A definition of a *Jordan scheme* may be obtained from the above one if we replace (A4) by

(J4) The linear span $\mathscr{A} = \langle A_0, A_1, ..., A_d \rangle$ is a $\star$-subalgebra of $M_X(\mathbb{C})$.

Each association scheme is a Jordan scheme, but the converse is not true as the following simple example shows

$$A_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, A_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Given a finite group G, one can construct an AS with $X = G$ and basic matrices $A_g, g \in G$ where A_g is a permutation matrix corresponding to the bijection $x \mapsto gx, x \in X$. This provides a one-to-one correspondence between finite groups and *thin* association schemes: those all basic matrices of which are permutational.

Motivated by this correspondence we classified all thin Jordan schemes and proved that they correspond to a special class of Moufang loops.

This and other results will be presented in the talk. It is a joint work with C. Pech and A. Woldar.

Spherical m-stiff configurations and related combinatorial structures

Hiroshi Nozaki*, Eiichi Bannai, Hirotake Kurihara

We investigate the existence of m-stiff configurations in the unit sphere S^{d-1}, which are spherical $(2m-1)$-designs that lie on m parallel hyperplanes. These configurations are occasionally found in tight spherical or Euclidean designs. We establish two non-existence results: (1) for each fixed integer $m > 5$, there exists no m-stiff configuration in S^{d-1} for sufficiently large d; (2) for each fixed integer $d > 10$, there exists no m-stiff configuration in S^{d-1} for sufficiently large m.

Some simple connections between doubly stochastic irreducible matrices and commutative association schemes

Safet Penjić*

Let $B \in \mathrm{Mat}_X(\mathbb{R})$ denote a normal irreducible nonnegative matrix, and $\mathscr{B} = \{p(B) \mid p \in \mathbb{C}[t]\}$ denote the vector space over $\mathbb{C}$ of all polynomials in B. Let us define a 01-matrix $\widehat{A}$ in the following way: $(\widehat{A})_{xy} = 1$ if and only if $(B)_{xy} > 0$ $(x, y \in X)$. Let $\Gamma = \Gamma(\widehat{A})$ denote a (di)graph with adjacency matrix $\widehat{A}$ and diameter D. We show some combinatorial properties of Γ under the assumption that $\mathscr{B}$ is the Bose–Mesner algebra of a commutative D-class association scheme.

This is work in progress.

q-Characters for quantum symmetric pairs

Tomasz Przezdziecki*

It is well known that quantum affine algebras admit three distinct presentations (Kac-Moody, new Drinfeld and RTT). Relatively recently, the same has been shown to hold for a broad family of quantum affine symmetric pairs (of which the q-Onsager algebra is a building block and rank 1 example). In particular, a Drinfeld-type presentation, due to Lu-Wang, is a new and exciting development. The focus of my talk will be the relationship between the usual Drinfeld presentation of quantum affine algebras and the Lu-Wang presentation of their coideal subalgebras. Remarkably, both presentations exhibit large commutative subalgebras, which are of particular interest to representation theory. More specifically, I will present several results concerning the properties of the generators of these commutative subalgebras, including their behaviour under inclusion and coproduct, as well as their spectra on finite-dimensional representations. These results will then be used to define an analogue of the q-character homomorphism for quantum symmetric pairs. I will compute the q-characters of evaluation modules, and discuss applications to categorification.

Elliptic hypergeometric functions and the Ruijsenaars model

Hjalmar Rosengren*

Much of Paul Terwilliger's work has been about relations between algebra, combinatorics and q-special functions, such as the polynomials in the Askey scheme. Many of these relations can or should be extended to a more general elliptic setting. In this talk, I will give a very brief introduction to elliptic special functions. This will be exemplified with recent work (joint with Eric Rains) on Q-operators for the Ruijsenaars model.

A uniform approach to the positive part of $U_q(\widehat{\mathfrak{sl}}_2)$ and generalized reflection equations

Chenwei Ruan*

The positive part U_q^+ of the quantized enveloping algebra $U_q(\widehat{\mathfrak{sl}}_2)$ is studied in both mathematics and mathematical physics. In the literature there are three PBW basis for U_q^+, due to Damiani, Beck, Terwilliger respectively. In the first part of this talk, we will present a uniform approach to the three PBW basis. In the second part, we will discuss an application of this uniform approach. This application is motivated by two recent results due to Baseilhac 2021 and Lemarthe, Beseilhac, Gainutdinov 2023. We will construct fused K-operators of spin-j in closed matrix form. As we will see, the fused K-operators satisfy generalized reflection equations.

Counting edges of different types in a local graph of a Grassmann graph

Ian Seong*

Let $\mathscr{V}$ denote a finite-dimensional vector space over a finite field. The corresponding projective geometry P is the poset consisting of the subspaces of $\mathscr{V}$, with partial order by inclusion. A Grassmann graph Γ associated with P is known to be distance-regular. Pick distinct vertices x, y of Γ that are not adjacent and at distance less than the diameter of Γ. Consider a two-vertex stabilizer $\mathrm{Stab}(x,y)$ in $GL(\mathscr{V})$. It is known that there are five orbits of the $\mathrm{Stab}(x,y)$-action on the local graph of x, denoted by $\Gamma(x)$. In this talk we define three types of edges in $\Gamma(x)$, namely type 0, type +, type -. For adjacent vertices $w, z \in \Gamma(x)$ such that w, z are equidistant from y, the type of the edge wz depends on the subspaces $w+z, w, z, w\cap z$ and their intersections with y. For each pair of orbits $\mathscr{O}, \mathscr{N}$ in $\Gamma(x)$, and a given vertex $w \in \mathscr{O}$, we find the number of vertices $z \in \mathscr{N}$ such that the edge wz has (i) type 0, (ii) type +, (iii) type -. To do this, we make heavy use of a subalgebra $\mathscr{H}$ of $\mathrm{Mat}_P(\mathbb{C})$ that contains some matrices that are closely related to the five orbits in $\Gamma(x)$.

2-homogeneous bipartite distance-regular graphs and the quantum group $U_q'(\mathfrak{so}_6)$

Paul Terwilliger*

We investigate a 2-homogeneous bipartite distance-regular graph Γ from an S_3-symmetric point of view. First assume that Γ is a hypercube. Together with Bill Martin, we recently used Γ to obtain an irreducible module for the Lie algebra $\mathfrak{sl}_4(\mathbb{C})$. It is known that $\mathfrak{sl}_4(\mathbb{C})$ is isomorphic to the special orthogonal Lie algebra $\mathfrak{so}_6(\mathbb{C})$. Next assume that Γ is not a hypercube. We use Γ to obtain an irreducible module for the nonstandard quantum group $U_q'(\mathfrak{so}_6)$ introduced by Gavrilik and Klimyk in 1991.

Almost amorphic association schemes

Edwin van Dam*, Jack Koolen, Yanzhen Xiong

In earlier work (JCTA 2025), we characterized amorphic association schemes in terms of fusions of pairs of relations, and showed among others that any d-class association scheme in which at least $d-1$-relations are strongly regular of Latin square type or negative Latin square type must be amorphic.

In this talk, we construct (non-amorphic) association schemes in which precisely $d-2$ relations are strongly regular of Latin square type (for any d).

Eigenspace embeddings of imprimitive association schemes

Janoš Vidali*

For a given symmetric association scheme $\mathscr{A}$ and its eigenspace S_j there exists a mapping of vertices of $\mathscr{A}$ to unit vectors of S_j, known as the spherical representation of $\mathscr{A}$ in S_j, such that the inner products of these vectors only depend on the relation between the corresponding vertices; furthermore, these inner products only depend on the parameters of $\mathscr{A}$. We consider parameters of imprimitive association schemes listed as open cases in the list of parameters for quotient-polynomial graphs recently published by Herman and Maleki, and study embeddings of their substructures into some eigenspaces consistent with spherical representations of the putative association schemes. Using this, we obtain nonexistence for two parameter sets for 4-class association schemes and one parameter sets for a 5-class association scheme passing all previously known feasibility conditions, as well as uniqueness for two parameter sets for 5-class association schemes.

CMV bispectrality

Luc Vinet*, Alexei Zhedanov

It will be shown that the Jacobi polynomials orthogonal on the unit circle (the Jacobi OPUC) are CMV bispectral. That is, the corresponding Laurent polynomials in the CMV basis satisfy two dual ordinary eigenvalue problems: a 5-term recurrence relation and a differential equation of Dunkl type. This is presumably the first nontrivial explicitly identified example of CMV bispectral OPUC. A circle Jacobi algebra will be introduced and seen to entail the fundamental properties of the Jacobi OPUC via its representations.

Multivariate Affine q-Krawtchouk Polynomials and Association Schemes over Galois Rings

Yuta Watanabe*

The first eigenmatrix (or character table) of the Hamming association scheme is classically described using Krawtchouk polynomials. Two major directions for generalizing these polynomials are well known: the multivariate extension and the q-analog. The

former appears, for example, in the context of ordered Hamming schemes, while the latter arises in the study of bilinear forms schemes, where the eigenmatrices are described by affine q-Krawtchouk polynomials. In this talk, we propose a further generalization that synthesizes these two directions. Specifically, we introduce a new class of multivariate affine q-Krawtchouk polynomials and demonstrate that they describe the first eigenmatrix of an association scheme defined on the space of matrices over Galois rings.

Codes and Designs in Classical Association Schemes

Charlene Weiss*

Classical association schemes essentially consist of the Hamming and Johnson schemes, along with their ordinary and affine q-analogs: the q-Johnson scheme, polar space schemes, and schemes on bilinear, alternating bilinear, and Hermitian forms. These schemes play an important role in combinatorics. First, they appear to form the core of (P and Q)-polynomial association schemes, as conjectured by Bannai in the 1970s. Second, many important codes and designs with broad applications beyond combinatorics are found within these classical schemes. We study such codes and designs, focusing on two key questions: How large can a code be? Do nontrivial designs exist? A powerful approach to addressing the first question is Delsarte's linear program. While determining the optimum of this linear program for the Hamming and Johnson schemes has remained open since the 1970s, we present the optimal solutions for most of the other classical association schemes. As a byproduct, we derive bounds on so-called intersecting sets, providing new proofs of several well-known results collectively known as Erdős–Ko–Rado-type theorems. Lastly, we investigate the existence of designs in the ordinary q-analog schemes.

Triangle-Free Classical Distance-Regular Graphs Containing a Pentagon

Chih-wen Weng*

We will survey recent progress in the study of classical distance-regular graphs whose intersection numbers satisfy $a_1 = 0$ and $a_2 \neq 0$. In conjunction with existing results, we arrive at the following two theorems.

Theorem A. There exists no classical distance-regular graph with intersection number $c_2 = 1$ and diameter $d \geq 4$.

Theorem B. A classical distance-regular graph with diameter $d \geq 4$ and intersection numbers $a_1 = 0$ and $a_2 \neq 0$ has parameters that fall into one of two possibilities: either $(d, b, \alpha, \beta) = (d, -2, -3, -1 - b^d)$ or $(d, -3, -2, (-1 - (-3)^d)/2)$. The first case corresponds to Hermitian forms graph $\mathrm{Her}_2(d)$, while the existence of a graph with the second set of parameters remains open.

Finite bivariate Tratnik functions

Nicolas Crampé, Meri Zaimi*

In the context of algebraic combinatorics, P- and Q-polynomial association schemes are important objects and are closely related to distance-regular graphs. The polynomials appearing in these structures are classified by Leonard's theorem, and they belong to the discrete part of the (q-)Askey scheme. Relatively recently, the notions of P- and Q-polynomial association schemes as well as of distance-regular graphs have been generalized to the multivariate case. There is however no multivariate analog of Leonard's theorem. With the purpose of progressing in that direction, I will discuss ongoing work concerning certain finite families of bivariate functions, said of Tratnik type, which are expressed as an intricate product of univariate polynomials of the (q-)Askey scheme. The goal is to classify such functions which satisfy some generalized bispectral properties, that is, two recurrence relations and two (q-)difference equations of certain types.

Quantum State Transfer in Association Schemes

Hanmeng Zhan*

Quantum walks are quantum analogues of random walks. One desired phenomenon in quantum walks is to transfer a state from one vertex to another with probability arbitrarily close to 1. In this talk, I will discuss such phenomenon in coin quantum walks, and show how properties of association schemes help us achieve state transfer in different families of graphs.

Multivariate P-polynomial association schemes and m-distance regular graphs

Xiaohong Zhang*

An association scheme is P-polynomial if and only if it consists of the distance matrices of a distance-regular graph. Recently, a generalization to multivariate P-polynomial association schemes has been proposed. In this talk, we will introduce m-distance-regular graphs and show their connection to multivariate P-polynomial association schemes.

From cyclotomic to orthogonal polynomials: Sturm meets Ramanujan

Alexei Zhedanov*

We demonstrate how the Sturm algorithm applied to cyclotomic polynomials leads to new families of orthogonal polynomials (both on the real line and on the unit circle). Corresponding mirror dual polynomials are related to the trigonometric Ramanujan sums.

Twin Buildings and Hypergroups, I

Christopher French, Paul-Hermann Zieschang*

The notion of a building is a far reaching and meaningful generalization of the concept of a projective plane. Some projective planes can be identified with groups. For instance, the Fano plane bears exactly the same information as the simple group $L_3(2)$ of order 168. More generally, the projective planes originating from finite dimensional vector spaces over fields can be constructed from the groups $L_n(F)$, F a field, and *vice versa*.

Not all buildings can be connected with groups in the above mentioned way. Tits' famous theorem on buildings of spherical type gives a sufficient condition for a building to be connected with a group in the above described fashion.

Here is where hypergroups come into the game. On the one hand, hypergroups (in the spirit of [1]) provide a far reaching and meaningful generalization of the notion of a group. On the other hand, they allow to generalize the above mentioned correspondence between certain classes of buildings and certain groups to *all* buildings and a well understood class of hypergroups (which we call Coxeter hypergroups). The proof of this observation is not too difficult; cf. [2].

For twin buildings, the situation seems to be more delicate; cf. [3]. We identified a specific class of hypergroups (which we call twin Coxeter hypergroups), each regular action of which determines a twin building. However, we do not know whether or not all twin buildings come do from a hypergroup action in the above sense. To fill this gap we need to settle a pure building theoretic question, a question which will be discussed in the subsequent talk.

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CONTRIBUTED TALKS

Classification of thin distance-regular graphs with classical parameters

Hong-Jun Ge*, Jack Koolen

In 1995, Terwilliger showed that each local graph Δ of a thin distance-regular graph Γ is co-edge-regular and has at most five distinct eigenvalues. Furthermore, Terwilliger demonstrated that the non-principal eigenvalues of Δ are contained in the set

$$\left\{\beta-\alpha-1, \alpha\frac{b^D-b}{b-1}-1, -1, -b-1\right\},$$

if Γ has classical parameters (D,b,α,β). To classify thin distance-regular graphs with classical parameters (D,b,α,β), we focus on co-edge-regular graphs with at most five distinct eigenvalues.

We will provide a classification of co-edge-regular graphs with smallest eigenvalue -3 and at most four distinct eigenvalues. As a consequence, we obtain a classification of thin distance-regular graphs with classical parameters $(D,2,\alpha,\beta)$.

Surprisingly, we also find a new infinite family of co-edge-regular graphs that are cospectral with certain clique-extensions of specific Latin Square graphs. This family demonstrates that the local graph of thin Q-polynomial distance-regular graphs may be more complicated than we hoped.

Codes over Finite Ring $\mathbb{Z}_k$, MacWilliams Identity and Theta Function

Fengxia Liu*

In this talk, I will introduce a paper that we recently completed. In this paper, we study linear codes over $\mathbb{Z}_k$ based on lattices and theta functions. We obtain the complete weight enumerators MacWilliams identity and the symmetrized weight enumerators MacWilliams identity based on the theory of theta function. We extend the main work by Bannai, Dougherty, Harada and Oura to the finite ring $\mathbb{Z}_k$ for any positive integer k and present the complete weight enumerators MacWilliams identity in genus g. When $k=p$ is a prime number, we establish the relationship between the theta function of associated lattices over a cyclotomic field and the complete weight enumerators with Hamming weight of codes, which is an analogy of the results by G. Van der Geer and F. Hirzebruch since they showed the identity with the Lee weight enumerators.

Cayley regularity graphs of full transformation semigroups

Nuttawoot Nupo*, Chollawat Pookpienlert, Yanisa Chaiya

Let X_n be a set of n elements and T_n the semigroup of full transformations on X_n under the composition of functions. Denote by $CR(T_n)$ the Cayley regularity graph of T_n which is a special digraph whose vertex set is T_n and arc set contains all ordered pairs $(\alpha,\beta) \in T_n \times T_n$ where $\alpha = \alpha\beta\alpha$. In this talk, we introduce basic comprehension and provide a characterization of arcs of $CR(T_n)$. Moreover, we present structural properties

of $CR(T_n)$ consisting of connectedness, completeness and traversability. We also propose the planarity of $CR(T_n)$ and present inequalities for certain invariant parameters of $CR(T_n)$.

Homomorphisms from the tetrahedron algebra $\boxtimes$ to the special orthogonal algebra $\mathfrak{so}_4$

Aaron Pagaygay*, John Vincent Morales

Let $\mathbb{F}$ denote an algebraically closed field with characteristic 0. The special orthogonal algebra $\mathfrak{so}_4$ is a six-dimensional $\mathbb{F}$-Lie algebra which belongs to the well-studied family of Lie algebras known as classical Lie algebras. A finite-dimensional $\mathbb{F}$-vector space W supports an $\mathfrak{so}_4$-module structure whenever there exists linear maps from W to W associated to the elements of $\mathfrak{so}_4$ such that the linear maps capture the defining bracket relations of $\mathfrak{so}_4$. Suppose V is an arbitrary finite-dimensional irreducible $\mathfrak{so}_4$-module. In this presentation, we obtain four tridiagonal pairs on V, and via these pairs, we display four Lie algebra homomorphisms from the tetrahedron algebra $\boxtimes$ to $\mathfrak{so}_4$. Consequently, we describe how the generators of $\boxtimes$ act on V. Finally, we show that V is isomorphic to a tensor product of two evaluation modules.

This is based on a joint work with John Vincent S. Morales.

Association Schemes from Matrices over Finite Fields

Andrew Mendelsohn, Christian Porter*

It is well known that association schemes may be defined on matrix rings over finite fields $M_n(\mathbb{F}_q)$ using the rank function: if the difference of two matrices has rank 0, place the pair into relation R_0; else place the pair into R_1. We extend this notion when $n = 2$ by separating R_1 into a number of subrelations using the matrix determinant: let R_0 be the diagonal relation (all pairs $(x,x) \in M_n(\mathbb{F}_q) \times M_n(\mathbb{F}_q)$) and let the ith relation be the pairs of distinct matrices with difference having determinant $i \in \mathbb{Z}/q\mathbb{Z}$, for $i = 1,2,...,q$. We then prove that we again obtain association schemes, denoted $\mathscr{C}(2,q)$. We show that considering the analogous definition for any $n > 2$, we do not even obtain coherent configurations. We then analyse the properties of these (translation) schemes: we compute the intersection numbers of $\mathscr{C}(2,q)$ for any prime q, prove that they are not P-polynomial or thin, and find relations between the intersection numbers. Finally, we show that $\mathscr{C}(2,3)$ is not Q-polynomial.

Tensor K-matrices: q-Onsager algebra and beyond

Bart Vlaar*, Andrea Appel

R-matrices, solutions of cubic braid relations in tensor products, arise naturally in representations of quasitriangular bialgebras. One can also consider (tensor) K-matrices, solutions of cylindrical (quartic) braid relations. They appear in a more refined algebraic context, involving a pair consisting of a quasitriangular bialgebra A and coideal subalgebra

B. The key family of examples is given by quantum symmetric pairs where A is a Drinfeld-Jimbo quantum group and B a Letzter-Kolb q-deformed fixed-point subalgebra. Quantum groups of affine type are particularly interesting, and a key example of a quantum symmetric pair is given by $A = U_q(\widehat{\mathfrak{sl}_2})$ and $B =$ (embedded) q-Onsager algebra.

In recent joint work with Andrea Appel we show that tensor K-matrices canonically yield ring homomorphisms from the Grothendieck ring of finite-dimensional A-modules to (a commutative subalgebra of) B. If A is a quantum group of affine type then the construction is valid upon extending scalars to formal series. It specializes to existing constructions in quantum integrable systems with boundaries, originally due to E. Sklyanin. Also, I outline possible future work towards universal descriptions of spectra of such systems (in the style of E. Frenkel and D. Hernandez) and analogues of the Frenkel-Mukhin-Reshetikhin q-characters.

A FEW WORDS ABOUT THE UNIVERSITY OF PRIMORSKA

Founded in 2003, the **University of Primorska (UP)** is the youngest of Slovenia's three public universities, serving over 5,600 students (23% of whom are international) from more than 45 countries. UP employs 782 staff, including 511 academic and 69 international faculty, and annually hosts over 270 international researchers and 165 doctoral students.

The University of Primorska (UP) consists of six faculties: the *Faculty of Mathematics, Natural Sciences, and Information Technologies (UP FAMNIT)*, the *Faculty of Education*, the *Faculty of Humanities*, the *Faculty of Management*, the *Faculty of Tourism Studies*, and the *Faculty of Health Sciences*. In addition, it includes the *Andrej Marušič Institute (UP IAM)*.

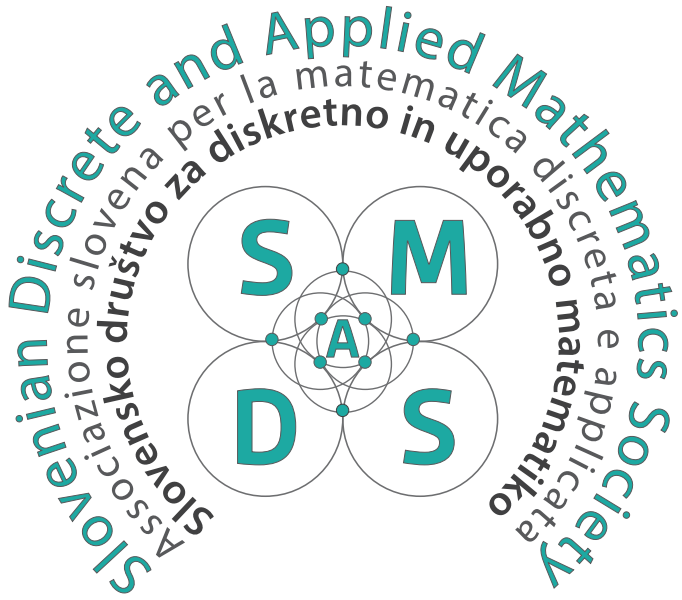
As a member of the *Transform4Europe* alliance and the *European Network of Innovative Higher Education Institutions (ENIHEI)*, the University of Primorska actively promotes inclusive and innovative education. In the 2024/25 academic year, it offers 79 study programmes, 17 of which are conducted in English, and annually organizes up to 10 scientific conferences along with several summer schools.

A key contributor to UP's academic excellence is **UP FAMNIT**, established in 2006. With 1035 students in 2024/25, it offers programmes in mathematics, computer science, data science, and related fields, many of which are taught in English. Together with UP IAM, it leads UP's research in mathematics, combinatorics, and theoretical computer science. Mathematical research at UP is especially strong in algebraic combinatorics and graph theory, where it has developed a global profile through high-impact publications and collaborations.

UP FAMNIT and UP IAM have positioned the university as a European hub for graph theory and combinatorics, organizing major events like the **8th European Congress of Mathematics (8ECM)** in Portorož (2021).

Through its research, education, and international engagement, UP fosters innovation, supports early-career researchers, and strengthens its role in regional and global academic communities.

A FEW WORDS ABOUT
SLOVENIAN DISCRETE AND APPLIED MATHEMATICS SOCIETY



The Slovenian Discrete and Applied Mathematics Society was established in Koper, Slovenia, on 14 December 2016. Its mission is to promote the mathematical sciences, with a particular focus on discrete and applied mathematics. The Society is research-oriented, actively publishing scientific literature and organizing academic events such as this one. Notably, it is involved in the publication of *Ars Mathematica Contemporanea* and *The Art of Discrete and Applied Mathematics*. The Society includes members, fellows, and honorary members.

A 'Member' may be any individual actively engaged in mathematical research, as evidenced in practice by authorship of a paper covered by *MathSciNet* or *Zentralblatt für Mathematitik*, or by enrollment in a research degree (and supported by a recommendation letter from the student's supervisor).

A 'Fellow' is a member who has strong international visibility and has made a positive impact on mathematics in Slovenia. Typically, a fellow would be expected to have at least 500 citations in the *MathSciNet* database, or be noted for some other achievements (such as an international award, or having given a keynote address in a large conference, or supervised a PhD student in or from Slovenia).

An 'Honorary Member' is an individual who has made outstanding contributions to the development of discrete or applied mathematics in Slovenia.

The Society has a Council to oversee its operations. It has a Nomination Committee, for nominating candidates for fellowship, and for considering candidates for honorary membership. Under the current rules, the Council of the Society will elect new Fellows and a limited number of Honorary Members at its annual meeting each year.

On 23 June 2018 the Council of the EMS approved unanimously the application of the Society for full membership of EMS. SDAMS is the first mathematics society from Eastern Europe that is a member of the EMS and does not cover only pure mathematics.

A FEW WORDS ABOUT
THE INSTITUTE OF MATHEMATICS, PHYSICS, AND MECHANICS

The Institute of Mathematics, Physics, and Mechanics (IMFM) is the central and largest scientific research institution in the field of mathematical sciences in the Republic of Slovenia. Founded on April 28, 1960, by the People's Assembly of the People's Republic of Slovenia, IMFM initially operated under the founding rights of the University of Ljubljana, to which they were transferred in 1972.

In 2003, the institute updated its legal registration to designate the Republic of Slovenia as its founder, effective retroactively from June 6, 1998. Despite this change, IMFM continued to function as a non-public institution, funding its research activities primarily through public calls by the Slovenian Research and Innovation Agency (ARIS), the Ministry of Defence, and through collaborations with industry partners.

The institute generally employs about 40 full-time researchers and collaborates with approximately 110 part-time researchers. It also plays an important role in educating young scientists, especially at the doctoral level, through partnerships with Slovenian universities.

On May 28, 2022, following a decision by the Government of the Republic of Slovenia, IMFM was formally transformed into a public research institution. This followed a ruling by the District Court in Ljubljana in 2014, which officially registered IMFM as a public institution with the Republic of Slovenia as its founder.

IMFM carries out research within four departments: Mathematics, Physics, Mechanics, and Theoretical Computer Science. It maintains active collaborations with prominent research institutions, including the Jožef Stefan Institute. In addition, IMFM researchers contribute to both teaching and research at various faculties of the University of Ljubljana, the University of Maribor, and the University of Primorska.

In collaboration with Slovenian universities, IMFM plays a vital role in training young researchers in mathematics, theoretical computer science, and physics. The institute is also a committed participant in several leading international research networks, advancing scientific knowledge on both national and global scales.

Combinatorics around the q-Onsager algebra

Kranjska Gora, Slovenia, June 22/23–27, 2025.

Koper, June 2025