WG 2024

Book of Abstracts

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WG 2024: Book of Abstracts

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List of contributions:

Plenary talks

Approximations of treewidth and other graph parameters

Hans L. Bodlaender (ToT Award)

Motivated from applications from graph algorithms and from sparse matrix factorization, in 1991, we (Bodlaender, Gilbert, Hafsteinsson and Kloks) introduced approximation algorithms for four different graph parameters, including treewidth and pathwidth. In this talk, the main ideas of these algorithms will be sketched, together with a discussion of the connections with sparse matrix factorization, and a historic overview of the further developments of approximations of these parameters.

Approximate Shortest Paths and Distance Oracles

Shiri Chechik (Invited Talk)

Computing shortest paths is one of the most fundamental graph problems. While classical shortest paths algorithms like Dijkstra's algorithm provide efficient solutions, they fall short when dealing with large graphs, e.g., continent-sized road networks, necessitating faster alternatives. Sublinear-time queries can be achieved through preprocessing, leading to the concept of distance oracles – data structures facilitating fast retrieval of distance estimates for any pair of nodes. Designing distance oracles involves balancing various parameters: construction time, data structure size, query time, and the accuracy of the estimated distance. Not only are distance oracles important structures by their own right with both practical and theoretical interest, but they also have strong ties to numerous other problems in algorithms, such as spanners, compact routing schemes and low-stretch spanning trees. A significant challenge arises from the dynamic nature of real-world networks, subject to both permanent changes (e.g., road additions) and temporary disruptions (e.g., closures or blockages). Dynamic settings capture permanent alterations, while fault tolerance settings address temporary changes. In this talk, I will explore key findings on approximate shortest paths and distance oracles across static, dynamic, and fault-tolerant settings.

Graph classes and logic

Michał Pilipczuk (Invited Talk)

Structural graph theory offers a wide range of notions that can be used to quantify the structure in graphs, such as various graph parameters, notions of embedding, and decompositions. The central concept here is that of a graph class, which is a set of graphs that share some common structural property of interest. But what if instead of relying on purely combinatorial definitions, we resorted to logic? That is, the idea is to define graph classes by excluding obstructions interpretable in logic, or by postulating the existence of some kind of decompositions definable in logic. During the talk we will both revisit, from this angle, classic results linking treewidth and cliquewidth with logic MSO, as well as survey the recent advances on understanding the combinatorics of monadically stable and monadically dependent graph classes, which correspond to the same questions asked for the First Order logic (FO).

Contributed talks

Fast winning strategies for the attacker in eternal domination

Guillaume Bagan, Nicolas Bousquet, Nacim Oijid and Théo Pierron

Dominating sets in graphs are often used to model some monitoring of the graph: guards are posted on the vertices of the dominating set, and they can thus react to attacks occurring on the unguarded vertices by moving there (yielding a new set of guards, which may not be dominating anymore). A dominating set is eternal if it can endlessly resist to attacks.

From the attacker's perspective, if we are given a non-eternal dominating set, the question is to determine how fast can we provoke an attack that cannot be handled by a neighboring guard. We investigate this question from a computational complexity point of view, by showing that this question is PSPACE-hard, even for graph classes where finding a minimum eternal dominating set is in P.

We then complement this result by giving polynomial time algorithms for cographs and trees, and showing a connection with tree-depth for the latter. We also investigate the problem from a parameterized complexity perspective, mainly considering two parameters: the number of guards and the number of steps.

Independent set reconfiguration in H-free graphs

Valentin Bartier, Nicolas Bousquet and Moritz Muehlenthaler

Given a graph G and two independent sets of G, the independent set reconfiguration problem asks whether one independent set can be transformed into the other by moving a single vertex at a time, such that at each intermediate step we have an independent set of G. We study the complexity of this problem for H-free graphs under the token sliding and token jumping rule. Our contribution is twofold. First, we prove a reconfiguration analogue of Alekseev's theorem, showing that the problem is PSPACE-complete unless H is a path or a subdivision of the claw. We then show that under the token sliding rule the problem admits a polynomial-time algorithm if the input graph is fork-free, generalizing known results for P_4-free graphs and claw-free graphs.

Enumerating Minimal Solution Sets for Metric Graph Problems

Benjamin Bergougnoux, Oscar Defrain and Fionn Mc Inerney

Problems from metric graph theory like Metric Dimension, Geodetic Set, and Strong Metric Dimension have recently had a strong impact in parameterized complexity by being the first known problems in NP to admit double-exponential lower bounds in the treewidth, and even in the vertex cover number for the latter. We initiate the study of enumerating minimal solution sets for these problems and show that they are also of great interest in enumeration. Specifically, we show that enumerating minimal resolving sets in graphs and minimal geodetic sets in split graphs are equivalent to enumerating minimal transversals in hypergraphs (denoted Trans-Enum), whose solvability in total-polynomial time is one of the most important open problems in algorithmic enumeration. This provides two new natural examples to a question that emerged in recent works: for which vertex (or edge) set graph property Π is the enumeration of minimal (or maximal) subsets satisfying Π equivalent to Trans-Enum? As very few properties are known to fit within this context—namely, those related to minimal domination—our results make significant progress in characterizing such properties, and provide new angles to approach Trans-Enum. In contrast, we observe that minimal strong resolving sets can be enumerated with polynomial delay. Additionally, we consider cases where our reductions do not apply, namely graphs with no long induced paths, and show both positive and negative results related to the enumeration and extension of partial solutions.

Feedback Vertex Set for pseudo-disk graphs in subexponential FPT time

Gaétan Berthe, Marin Bougeret, Daniel Gonçalves and Jean-Florent Raymond

In this paper we investigate the Feedback Vertex Set problem (FVS). This problem asks for a graph G and an integer k, if there is a set X of at most k vertices such that $G-X$ is acyclic. Such subexponential FPT algorithms are known to exist in planar and even H-minor free graphs from bidimensionality theory [Demaine et al. 2005], and there is a recent line of work lifting these results to geometric graph classes consisting of intersection of "fat" objects ([Fomin et al. 2012], [Grigoriev et al. 2014], [Lokshtanov et al. 2022], [An et al. 2023]). Here, we consider pseudo-disk graphs (a generalization of disk graphs). Assuming the ETH, for this class, FVS cannot be solved in time $2^{o(\sqrt{n})}$. We provide an algorithm running in time $2^{O(k^{9/10} logk)} n^{O(1)}$.

Oriented trees in $O(k\sqrt{k})$-chromatic digraphs, a subquadratic bound for Burr's conjecture

Stéphane Bessy, Daniel Gonçalves and Amadeus Reinald

In 1980, Burr conjectured that every directed graph with chromatic number $2k-2$ contains any oriented tree of order k as a subgraph. Burr showed that chromatic number $(k-1)^2$ suffices, which was improved in 2013 to $\frac{k^2}{2} - \frac{k}{2} + 1$ by Addario-Berry et al.

We give the first subquadratic bound for Burr's conjecture, by showing that every directed graph with chromatic number $8\sqrt{\frac{2}{15}}k\sqrt{k} + O(k)$ contains any oriented tree of order k. Moreover, we provide improved bounds of $\sqrt{\frac{4}{3}}k\sqrt{k} + O(k)$ for arborescences, and $(b-1)(k-3)+3$ for paths on b blocks, with $b \geq 2$.

Improved Outerplanarity Bounds for Planar Graphs

Therese Biedl and Debajyoti Mondal

In this paper, we study the outerplanarity of planar graphs, i.e., the number of times that we must (in a planar embedding that we can initially freely choose) remove the outerface vertices until the graph is empty. It is well-known that there are n-vertex graphs with outerplanarity $\frac{n}{6} + \Theta(1)$, and not difficult to show that the outerplanarity can never be bigger. We give here improved bounds of the form $\frac{n}{2g} + 2g + O(1)$, where g is the *fence-girth*, i.e., the length of the shortest cycle with vertices on both sides. This parameter g is at least the connectivity of the graph, and often bigger; for example, our results imply that planar bipartite graphs have outerplanarity $\frac{n}{8} + O(1)$. We also show that the outerplanarity of a planar graph G is at most $\frac{1}{2}\mathrm{diam}G + O(\sqrt{n})$, where $\mathrm{diam}G$ is the diameter of the graph. All our bounds are tight up to smaller-order terms, and a planar embedding that achieves the outerplanarity bound can be found in linear time.

XNLP-hardness of Parameterized Problems on Planar Graphs

Hans Bodlaender and Krisztina Szilagyi

The class XNLP consists of (parameterized) problems that can be solved nondeterministically in $f(k)n^{O(1)}$ time and $g(k)\log(n)$ space, where n is the size of the input instance and k the parameter. The class XALP consists of problems that can be solved in the above time and space with access to an additional stack. These two classes are a "natural home" for many standard graph problems and their generalizations.

In this paper, we show the hardness of several problems on planar graphs, parameterized by outerplanarity, treewidth and pathwidth, thus strengthening several existing results. In particular, we show XNLP-hardness of the following problems parameterized by outerplanarity: All-or-Nothing Flow, Target Outdegree Orientation, Capacitated (Red-Blue) Dominating Set, Target Set Selection etc. We also show the XNLP-completeness of Scattered Set parameterized by pathwidth and XALP-completeness parameterized by treewidth and outerplanarity.

Augmenting Plane Straight-Line Graphs to Meet Parity Constraints

Aleksander Bjoern Grodt Christiansen, Linda Kleist, Irene Parada and Eva Rotenberg

Given a plane geometric graph G on n vertices, we want to augment it so that given parity constraints of the vertex degrees are met. In other words, given a set R of vertices, we are interested in a plane geometric supergraph G' such that exactly the vertices of R have odd degree in $G' \setminus G$. We show that the question whether such a supergraph exists can be decided in polynomial time for two interesting cases. First, when the vertices are in convex position, we present a linear-time algorithm. Building on this insight, we solve the case when G is a plane geometric path. This solves an open problem posed by Catana, Olaverri, Tejel, and Urrutia (Appl. Math. Comput. 2020).

Covering a Graph with Minimal Local Sets

Nathan Claudet and Simon Perdrix

Local sets, a graph structure invariant under local complementation, have been originally introduced in the context of quantum computing for the study of quantum entanglement within the so-called graph state formalism. A local set in a graph is made of a non-empty set of vertices together with its odd neighborhood. We show that any graph can be covered by minimal local sets, i.e. that every vertex is contained in at least one local set that is minimal by inclusion. More precisely, we introduce an algorithm for finding a minimal local set cover in polynomial time. This result is proved by exploring the link between local sets and cut-rank. We prove some additional results on minimal local sets: we give tight bounds on their size, and we show that there can be exponentially many of them in a graph. Finally, we provide an extension of our definitions and our main result to q-multigraphs, the graphical counterpart of quantum qudit graph states.

A New Approach for Approximating Directed Rooted Networks

Sarel Cohen, Lior Kamma and Aikaterini Niklanovits

We consider the *k-outconnected directed Steiner tree* problem (k-DST). Given a directed edge-weighted graph $G = (V, E, w)$, where $V = \{r\} \cup S \cup T$, and an integer k, the goal is to find a minimum cost subgraph of G in which there are k edge-disjoint rt-paths for every terminal $t \in T$. This is one of the most fundamental problems in theoretical computer science, and has many applications in the design of practical networks.

The problem is known to be NP-Hard. Furthermore, Cheriyan et al. (2014) showed that k-DST is at least as hard to approximate as the label cover problem, which, in turn, implies that there is no $2^{\log^{1-\varepsilon}(|V|)}$-approximation for k-DST for any fixed $\varepsilon > 0$ unless $\mathsf{NP} \subseteq \mathsf{DTIME}(n^{\mathsf{polylog}(n)})$. A result by Nelson (2007) implies that for every $c < \frac{1}{2}$, k-DST has no polynomial-time $2^{\log^{1-\delta_c(|S|)}(|S|)}$-approximation, unless SAT on m variables can be decided in time $2^{O(2^{\log^{1-\delta_c(m)}(m)})}$, where $\delta_c(m) = 1 - \frac{1}{(\log\log m)^c}$. The question on whether a polynomial time, subpolynomial approximation algorithm exists for k-DST was answered negatively by Grandoni et al. (2018), by proving an approximation hardness of $\Omega(|T|/\log|T|)$ under $\mathsf{NP} \neq \mathsf{ZPP}$.

In this paper, inspired by modern day applications, we focus on developing efficient algorithms for k-DST when the majority of nodes are terminals. Such applications of the k-DST problem are multicast routing for Netflix or Youtube servers, as they present us with networks in which T constitutes almost the entire network, and moreover the terminals do not transmit any data. In this paper we provide the first approximation algorithm for k-DST on such graphs, in which the approximation ratio depends (primarily) on the size of S. We present a randomized algorithm that finds a solution of weight at most $O(k|S|\log|T|)$ times the optimal weight, and with high probability runs in polynomial time.

Approximations and Hardness of Packing Partially Ordered Items

Ilan Doron-Arad, Guy Kortsarz, Joseph Naor, Baruch Schieber and Hadas Shachnai

Motivated by applications in production planning and storage allocation in hierarchical databases, we initiate the study of covering partially ordered items (CPO). Given a capacity $k \in \mathbb{Z}^+$, and a directed graph $G = (V, E)$ where each vertex has a size in $\{0, 1, \ldots, k\}$, we seek a collection of subsets of vertices $S_1, \ldots, S_m$ that cover all the vertices, such that for any $1 \le j \le m$, the total size of vertices in S_j is bounded by k, and there are no edges from $V \setminus S_j$ to S_j. The objective is to minimize the number of subsets m. CPO is closely related to the rule caching problem (RCP) that is of wide interest in the networking area. The input for RCP is a directed graph $G = (V, E)$, a profit function $p : V \to \mathbb{Z}_0^+$, and $k \in \mathbb{Z}^+$. The output is a subset $S \subseteq V$ of maximum profit such that $|S| \le k$ and there are no edges from $V \setminus S$ to S.

Our main result is a 2-approximation algorithm for CPO on out-trees, complemented by an asymptotic 1.5-hardness of approximation result. We also give a two-way reduction between RCP and the densest k-subhypergraph problem, surprisingly showing that the problems are equivalent w.r.t. polynomial-time approximation within any factor $\rho \ge 1$. This implies that RCP cannot be approximated within factor $|V|^{1-\varepsilon}$ for any fixed $\varepsilon > 0$, under standard complexity assumptions. Prior to this work, RCP was just known to be strongly NP-hard. We further show that there is no EPTAS for the special case of RCP where the profits are uniform, assuming Gap-ETH. Since this variant admits a PTAS, we essentially resolve the complexity status of this problem.

Beyond recognizing well-covered graphs

Carl Feghali, Malory Marin and Rémi Watrigant

We prove a number of results related to the computational complexity of recognizing well-covered graphs. For instance, let k and s be positive integers and G be a graph. Then G is said

- W_k if for any k pairwise disjoint independent vertex sets $A_1, \ldots, A_k$ in G, there exist k pairwise disjoint maximum independent sets $S_1, \ldots, S_k$ in G such that $A_i \subseteq S_i$ for $i \subseteq [k]$.
- E_s if every independent set in G of size at most s is contained in a maximum independent set in G.

Chvatal and Slater (1993) and Sankaranarayana and Stewart (1992) famously showed that recognizing W_1 graphs or, equivalently, well-covered graphs is coNP-complete. We extend this result by showing that recognizing W_{k+1} graphs in either W_k or E_s graphs is coNP-complete. This answers a question of Levit and Tankus (2023) and strengthens a theorem of Feghali and Marin (2024). We also show that recognizing E_{s+1} graphs is θ_p^2-complete even in E_s graphs, where $\theta_p^2 = P^{NP[log]}$ is the class of problems solvable in polynomial time using a logarithmic number of calls to a Sat oracle. This strengthens a theorem of Berge, Busson, Feghali and Watrigant (2023). We also obtain the complete picture of the complexity of recognizing chordal W_k and E_k graphs which, in particular, simplifies and generalizes a result of Dettlaff, Henning and Topp (2023).

Untangling Gaussian Mixtures

Eva Fluck, Sandra Kiefer and Christoph Standke

Tangles were originally introduced as a concept to formalize regions of high connectivity in graphs. In recent years, they have also been discovered as a link between structural graph theory and data science: when interpreting similarity in data sets as connectivity between points, finding clusters in the data essentially amounts to finding tangles in the underlying graphs.

This paper explores the potential of tangles in data sets as a means for a formal quantitative study of clusters. Real-world data often follow a normal distribution. Accounting for this, we develop a quantitative theory of tangles in data sets drawn from Gaussian mixtures. To this end, we equip the data with a graph structure that models similarity between the points and allows us to apply tangle theory. We provide explicit conditions under which tangles associated with the marginal Gaussian distributions exist asymptotically almost surely. This can be considered a sufficient formal criterion for the separabability of clusters in the data.

Face-hitting Dominating Sets in Planar Graphs

P. Francis, Abraham M. Illickan, Lijo M. Jose and Deepak Rajendraprasad (Best Student Paper Award)

A dominating set of a graph G is a subset S of its vertices such that each vertex of G not in S has a neighbor in S. A face-hitting set of a plane graph G is a set T of vertices in G such that every face of G contains at least one vertex of T. We show that the vertex-set of every plane graph without isolated vertices and 2-length faces can be partitioned into two disjoint sets so that both the sets are dominating and face-hitting. Matheson and Tarjan [European J. Combin., 1996] conjectured that every plane triangulation with a sufficiently large number of vertices n, has a dominating set of size at most $n/4$. Currently, the best known general bound for this is by Christiansen, Rotenberg and Rutschmann [SODA, 2024] who showed that every plane triangulation on $n > 10$ vertices has a dominating set of size at most $2n/7$. As a corollary of our main result, we improve their bound for n-vertex plane triangulations which contain a maximal independent set of size either less than $2n/7$ or more than $3n/7$.

The Parameterized Complexity Landscape of the Unsplittable Flow Problem

Robert Ganian, Mathis Rocton and Daniel Unterberger

We study the well-established problem of finding an optimal routing of unsplittable flows in a graph. While by now there is an extensive body of work targeting the problem on graph classes such as paths and trees, we aim at using the parameterized paradigm to identify its boundaries of tractability on general graphs. We develop novel algorithms and lower bounds which result in a full classification of the parameterized complexity of the problem with respect to natural structural parameterizations for the problem - notably maximum capacity, treewidth, maximum degree, and maximum flow length. In particular, we obtain a fixed-parameter algorithm for the problem when parameterized by all four of these parameters, establish XP-tractability as well as W[1]-hardness with respect to the former three and latter three parameters, and all remaining cases remain paraNP-hard.

Lightweight Near-Additive Spanners for Weighted Graphs

Yuval Gitlitz, Ofer Neiman and Richard Spence

An (α, β)-spanner of a weighted graph $G = (V, E)$, is a subgraph H such that for every $u, v \in V$, $d_H(u, v) \leq \alpha \cdot d_G(u, v) + \beta$. The main parameters of interest for spanners are their size (number of edges) and their lightness (the ratio between the total weight of H to the weight of a minimum spanning tree).

In this paper we focus on near-additive spanners, where $\alpha = 1 + \varepsilon$ for arbitrarily small $\varepsilon > 0$. We show the first construction of *light* spanners in this setting. Specifically, for any integer parameter $k \geq 1$, we obtain an $(1+\varepsilon, O(k/\varepsilon)^k \cdot W(\cdot, \cdot))$-spanner, where $W(\cdot, \cdot)$ indicates for every pair $u, v \in V$ the heaviest edge in some shortest path between u, v, with lightness $\widetilde{O}(n^{1/k})$. In addition, we can also bound the number of edges in our spanner by $O(kn^{1+3/k})$.

Classification of finite directed vertex-colored ultrahomogeneous graphs

Irene Heinrich, Eda Kaja and Pascal Schweitzer

A binary relational structure R is ultrahomogeneous if every isomorphism of induced substructures of R extends to an automorphism of R. We classify the ultrahomogeneous finite binary relational structures with one binary relation and arbitrary many unary relations. In other words, we classify the finite vertex-colored directed ultrahomogeneous graphs. The classification comprises several general methods with which directed graphs can be combined or extended to create new ultrahomogeneous graphs. Together with explicitly given exceptions, we obtain exactly all vertex-colored directed ultrahomogeneous graphs this way. Our main technique is a technical tool that characterizes precisely under which conditions two binary relational structures with disjoint unary relations can be combined to form a larger ultrahomogeneous structure.

Recognition of Unit Segment and Polyline Graphs is ∃R-Complete

Michael Hoffmann, Tillmann Miltzow, Simon Weber and Lasse Wulf

Given a set of objects O in the plane, the corresponding intersection graph is defined as follows. A vertex is created for each object and an edge joins two vertices whenever the corresponding objects intersect. We study here the case of unit segments and polylines with exactly k bends. In the recognition problem, we are given a graph and want to decide whether the graph can be represented as the intersection graph of certain geometric objects. In previous work it was shown that various recognition problems are ∃R-complete, leaving unit segments and polylines as few remaining natural cases. We show that recognition for both families of objects is ∃R-complete.

Roman Cycle Hitting Set

Satyabrata Jana, Sounak Modak, Saket Saurabh and Kushal Singanporia

The variant of Dominating Set known as Roman Domination has been extensively studied in the fields of graph theory and graph algorithms. Fernau and Mann [arXiv:2302.11417] introduced the concept of Roman Vertex Cover and developed a fixed-parameter tractable (FPT) algorithm for the problem with running time of $\mathcal{O}^*(2^k)$, where k is the solution size. Building on this work, we investigate the parameterized algorithms for Roman Feedback Vertex Set (RFVS) and Roman Odd Cycle Transversal (ROCT), and demonstrate that they are also FPT. Our approach to RFVS is similar to the best-known algorithm for the non-Roman variant. However, the algorithm for ROCT is significantly different and utilizes the technique of recursive understanding and structural insights on unbreakable graphs.

Popular Solutions for Optimal Matchings

Telikepalli Kavitha

Let G be a bipartite graph where every vertex has a strict preference order over its neighbors. The preferences of a vertex over its neighbors extend naturally to preferences over matchings. A matching M is popular in G if there is no matching N such that vertices that prefer N outnumber those that prefer M. Every stable matching is popular. We consider the following variant: edges in G have utilities and it is only max-utility matchings that are relevant for us. We show there always exists a max-utility matching that is popular within the set of all max-utility matchings and such a matching can be efficiently computed. We focus on largest max-utility matchings and show a compact extended formulation for the polytope of largest max-utility matchings that are popular within the set of all largest max-utility matchings.

Degreewidth on Semi-Complete Digraphs

Ryan Keeney and Daniel Lokshtanov (Best Student Paper Award)

For a digraph G and ordering of G, the degreewidth of the ordering is the maximum number of backward edges incident to any vertex of G. The degreewidth $D(G)$ of G is defined as the minimum degreewidth of an ordering of G. A digraph G is semi-complete if every pair of vertices is connected by at least one edge, oriented if every pair of vertices is connected by at most one edge, and a tournament if every pair of vertices is connected by exactly one edge. We give a fixed parameter tractable (FPT) algorithm, with running time $D(G)^{O(D(G))}n + O(n^2)$, to compute the degreewidth of semi-complete digraphs. We then show that both the Feedback Arc Set and Cutwidth problems on semi-complete digraphs admit algorithms with running time $D(G)^{O(D(G))}n+O(n^2)$. Our algorithms resolve in the affirmative two open problems of Davot et al. [WG 2023], who asked whether there exists an FPT algorithm to compute the degreewidth of a tournament, and whether Feedback Arc Set on tournaments admits an FPT algorithm when parameterized by the degreewidth of the input digraph. Additionally, extending an argument of Davot et al. [WG 2023], we show that sorting by in-degree is a 3-approximation algorithm for degreewidth on semi-complete digraphs. Finally we prove that it is NP-hard to determine whether a given oriented digraph has degreewidth at most 2.

On the Connectivity of the Flip Graph of Plane Spanning Paths

Linda Kleist, Peter Kramer and Christian Rieck

Flip graphs of non-crossing configurations in the plane are widely studied objects, e.g., flip graph of triangulations, spanning trees, Hamiltonian cycles, and perfect matchings. Typically, it is an easy exercise to prove connectivity of a flip graph. In stark contrast, the connectivity of the flip graph of plane spanning paths on point sets in general position has been an open problem for more than 16 years.

In order to provide new insights, we investigate certain induced subgraphs. Firstly, we provide tight bounds on the diameter and the radius of the flip graph of spanning paths on points in convex position with one fixed endpoint. Secondly, we show that so-called suffix-independent paths induce a connected subgraph. Consequently, to answer the open problem affirmatively, it suffices to show that each path can be flipped to some suffix-independent path. Lastly, we investigate paths where one endpoint is fixed and provide tools to flip to suffix-independent paths. We show that these tools are strong enough to show connectivity of the flip graph of plane spanning paths on point sets with at most two onion layers.

Graph Reconstruction with Connectivity Queries

Kacper Kluk, Hoang La and Marta Piecyk

We study a problem of reconstruction of connected graphs where the input gives all subsets of size k that induce a connected subgraph. Originally introduced by Bastide et al. (WG 2023) for triples ($k = 3$), this problem received comprehensive attention in their work, alongside a study by Qi, who provided a complete characterization of graphs uniquely reconstructible via their connected triples, i.e. no other graphs share the same set of connected triples. Our contribution consists in output-polynomial time algorithms that enumerate every triangle-free graph (resp. every graph with bounded maximum degree) that is consistent with a specified set of connected k-sets. Notably, we prove that triangle-free graphs are uniquely reconstructible, while graphs with bounded maximum degree that are consistent with the same k-sets share a substantial common structure, differing only locally. We suspect that the problem is NP-hard in general and provide a NP-hardness proof for a variant where the connectivity is specified for only some k-sets (with k at least 4).

On polynomial kernelization for Stable Cutset

Stefan Kratsch and Van Bang Le

A stable cutset in a graph G is a set $S \subseteq V(G)$ such that vertices of S are pairwise non-adjacent and such that $G - S$ is disconnected, i.e., it is both stable (or independent) set and a cutset (or separator). Unlike general cutsets, it is NP-complete to determine whether a given graph G has any stable cutset. Recently, Rauch et al. [FCT 2023] gave a number of fixed-parameter tractable (FPT) algorithms, time $f(k) \cdot |V(G)|^c$, for Stable Cutset under a variety of parameters k such as the size of a (given) dominating set, the size of an odd cycle transversal, or the deletion distance to P_5-free graphs. Earlier works imply FPT algorithms relative to clique-width and relative to solution size.

We complement these findings by giving the first results on the existence of polynomial kernelizations for Stable Cutset, i.e., efficient preprocessing algorithms that return an equivalent instance of size polynomial in the parameter value. Under the standard assumption that $\mathsf{NP} \not\subseteq \mathsf{coNP/poly}$, we show that no polynomial kernelization is possible relative to the deletion distance to a single path, generalizing deletion distance to various graph classes, nor by the size of a (given) dominating set. We also show that under the same assumption no polynomial kernelization is possible relative to solution size, i.e., given (G, k) answering whether there is a stable cutset of size at most k. On the positive side, we show polynomial kernelizations for parameterization by modulators to a single clique, to a cluster or a co-cluster graph, and by twin cover.

Revisiting Path Contraction and Cycle Contraction

R. Krithika, Kutty Malu V K and Prafullkumar Tale

The Path Contraction and Cycle Contraction problems take as input an undirected graph G with n vertices, m edges and an integer k and determine whether one can obtain a path or a cycle, respectively, by performing at most k edge contractions in G. We revisit these NP-complete problems and prove the following results.

- Path Contraction admits an algorithm running in $\mathcal{O}^*(2^k)$ time. This improves over the current algorithm known for the problem [Algorithmica 2014].
- Cycle Contraction admits an algorithm running in time $\mathcal{O}^*((2 + \epsilon_\ell)^k)$ where $0 < \epsilon_\ell \leq 0.5509$ and ϵ_ℓ is inversely proportional to $\ell = n - k$.

Crucial to these results is an algorithm for a more general variant of Path Contraction, namely, Path Contraction With Constrained Ends. We also give an $\mathcal{O}^*(2.5191^n)$-time algorithm to solve the optimization version of Cycle Contraction.

Next, we turn our attention to complexity of the problems on restricted graph classes and show the following results.

- Path Contraction on planar graphs admits a polynomial-time algorithm using the known polynomial-time algorithm for Cycle Contraction (J. Graph Theory 1999) as a subroutine.
- Path Contraction on chordal graphs does not admit an algorithm running in time $\mathcal{O}(n^{2-\epsilon} \cdot 2^{o(tw)})$ for any $\epsilon > 0$, unless the Orthogonal Vectors Conjecture fails. Here, tw is the treewidth of the input graph.

The second result complements the $\mathcal{O}(nm)$-time, i.e., $\mathcal{O}(n^2 \cdot tw)$-time, algorithm known for the problem [Discret. Appl. Math. 2014].

On the Complexity of Simultaneous Geometric Embedding for Edge-Disjoint Graphs

Benedikt Künzel and Jonathan Rollin

Simultaneous Geometric Embedding (SGE) asks whether, for a given collection of graphs on the same vertex set V, there is an embedding of V in the plane that admits a crossing-free drawing with straightline edges for each of the given graphs. It is known that SGE is $\exists\mathbb{R}$-complete, that is, the problem is polynomially equivalent to deciding whether a system of polynomial equations and inequalities with integer coefficients has a real solution. We prove that SGE remains $\exists\mathbb{R}$-complete for edge-disjoint input graphs, that is, for collections of graphs without so-called public edges. As an intermediate result, we prove that it is $\exists\mathbb{R}$-complete to decide whether a directional walk without repeating edges is realizable. Here, a directional walk consists of a sequence of not-necessarily distinct vertices (a walk) and a function prescribing for each inner position whether the walk shall turn left or shall turn right. A directional walk is realizable, if there is an embedding of its vertices in the plane such that the embedded walk turns according to the given directions. Previously it was known that realization is $\exists\mathbb{R}$-complete to decide for directional walks repeating each edge at most 336 times. This answers two questions posed by Schaefer ["On the Complexity of Some Geometric Problems With Fixed Parameters", JGAA 2021].

Many views of planar point sets

Jan Kynčl and Jan Soukup (Best Paper Award)

Given a set P of n points in the plane and two points x and y not in P, such that their union is in general position, we say that x and y have the same view of P if the points of P are visible in the same cyclic order from x and y. We show that for every set P of n points in general position in the plane, there are $\Omega(n^4)$ points with mutually distinct views of P, confirming a conjecture by Diaz-Banez, Fabila-Monroy and Perez-Lantero and a conjecture by Bieri and Schmidt. We also provide an easier alternative proof for point sets in strong general position.

Finding d-Cuts in Graphs of Bounded Diameter, Graphs of Bounded Radius and H-Free Graphs

Felicia Lucke, Ali Momeni, Daniel Paulusma and Siani Smith

The d-Cut problem is to decide if a graph has an edge cut such that each vertex has at most d neighbours at the opposite side of the cut. If $d = 1$, we obtain the intensively studied Matching Cut problem. The d-Cut problem has been studied as well, but a systematic study for special graph classes was lacking. We initiate such a study and consider classes of bounded diameter, bounded radius and H-free graphs. We prove that for all $d \geq 2$, d-Cut is polynomial-time solvable for graphs of diameter 2, $(P_3 + P_4)$-free graphs and P_5-free graphs. These results extend known results for $d = 1$. However, we also prove several NP-hardness results for d-Cut that contrast known polynomial time results for $d = 1$. Our results lead to full dichotomies for bounded diameter and bounded radius and to partial dichotomies for H-free graphs; for $d \geq 3$, our classification of d-Cut for H-free graphs only has three open cases.

Exact and approximate k-planarity testing for maximal graphs of small pathwidth

Miriam Münch, Maximilian Pfister and Ignaz Rutter

A graph is k-planar, if it admits a drawing with at most k crossings per edge. Testing whether a given graph is k-planar is known to be NP-complete. For $k = 1$ the problem remains NP-complete even for graphs of bounded pathwidth. In this paper we give linear-time algorithms for efficiently testing 1-planarity of maximal pathwidth 3− and maximal pathwidth 4-graphs. Closely related to the concept of k-planarity is the local crossing number of a graph G; i.e. the minimum number k such that there is a drawing of G in which no edge has more than k crossings.For general graphs of pathwidth 3 we give a 7-approximation for the local crossing number. Finally we employ a technique used by Biedl et al. to derive an $O(\omega)$-approximation for maximal pathwidth ω-graphs.

The Complexity of Diameter on H-free graphs

Jelle Oostveen, Daniel Paulusma and Erik Jan van Leeuwen

The intensively studied Diameter problem is to find the diameter of a given connected graph. We investigate, for the first time in a structured manner, the complexity of Diameter for H-free graphs, that is, graphs that do not contain a fixed graph H as an induced subgraph. We first show that if H is not a linear forest with small components, then Diameter cannot be solved in subquadratic time for H-free graphs under SETH. For some small linear forests, we do show linear-time algorithms for solving Diameter. For other linear forests H, we make progress towards linear-time algorithms, by considering specific diameter values. If H is a linear forest, the maximum value of the diameter of any graph in a connected H-free graph class is some constant $d_{\max}$ dependent only on H. We give linear-time algorithms for deciding if a connected H-free graph has diameter $d_{\max}$, for several linear forests H. In contrast, for one such linear forest H, Diameter cannot be solved in subquadratic time for H-free graphs under SETH. Moreover, we even show that, for several other linear forests H, one cannot decide in subquadratic time if a connected H-free graph has diameter $d_{\max}$ under SETH.

Approximating branchwidth on parametric extensions of planarity

Dimitrios Thilikos and Sebastian Wiederrecht

The branchwidth of a graph has been introduced by Roberson and Seymour as a measure of the tree-decomposability of a graph, alternative to treewidth. Branchwidth is polynomially computable on planar graphs by the celebrated "Ratcatcher"-algorithm of Seymour and Thomas. We investigate an extension of this algorithm to minor-closed graph classes, further than planar graphs, as follows: Let H_1 be a graph embeddable in the torus and H_2 be a graph embeddable in the projective plane. We prove that every $\{H_1, H_2\}$-minor free graph G contains a subgraph G' where the difference between the branchwidth of G and the branchwidth of G' is bounded by some constant, depending only on H_1 and H_2. Moreover, the graph G admits a tree decomposition where all torsos are planar. This decomposition can be used for deriving an constant-additive approximation for branchwidth: For $\{H_1, H_2\}$-minor free graphs, there is a constant c (depending on H_1 and H_2) and an $O(|V(G)|^3)$-time algorithm that, given a graph G, outputs a value b such that the branchwidth of G is between b and $b + c$.

WG 2024 - PROGRAM

Time	Wednesday, June 19	Thursday, June 20	Friday, June 21
07:00	Room Rute available for discussions and work in small groups	Room Rute available for discussions and work in small groups	Room Rute available for discussions and work in small groups
08:30	Registration	Registration	Registration
08:50	**Welcome**		
09:00	*chair: S. Cabello* Invited talk **Shiri Chechik** Approximate Shortest Paths and Distance Oracles	*chair: D. Paulusma* ToT award talk **Hans L. Bodlaender** Approximations of Treewidth and Other Graph Parameters	*chair: D. Kráľ* Invited talk **Michał Pilipczuk** Graph Classes and Logic
09:50	**Ilan Doron-Arad:** Approximations and Hardness of Packing Partially Ordered Items	**Fionn Mc Inerney**: Enumerating Minimal Solution Sets for Metric Graph Problems	**Moritz Muehlenthaler**: Independent Set Reconfiguration in H-free Graphs
10:10	**Telikepalli Kavitha:** Popular Solutions for Optimal Matchings	**Malory Marin**: Beyond Recognizing Well-Covered Graphs	**Peter Kramer**: On the Connectivity of the Flip Graph of Plane Spanning Paths
10:30	coffee break	coffee break	coffee break
11:00	*chair: P. Kindermann* **Gaétan Berthe**: Feedback Vertex Set for pseudo-disk graphs in subexponential FPT time	*chair: E. Köhler* **Aikaterini Niklanovits**: A New Approach for Approximating Directed Rooted Networks	*chair: M. Krnc* **Krisztina Szilagyi**: XNLP-hardness of Parameterized Problems on Planar Graphs
11:20	**Irene Parada**: Augmenting Plane Straight-Line Graphs to Meet Parity Constraints	**Eva Fluck**: Untangling Gaussian Mixtures	**Debajyoti Mondal**: Improved Outerplanarity Bounds for Planar Graphs
11:40	**Simon Weber**: Recognition of Unit Segment and Polyline Graphs is ∃R-Complete	**Yuval Gitlitz**: Lightweight Near-Additive Spanners for Weighted Graphs	**Miriam Münch**: Exact and Approximate k-planarity Testing for Maximal Graphs of Small Pathwidth
12:00	**Benedikt Künzel**: On the Complexity of Simultaneous Geometric Embedding for Edge-Disjoint Graphs	**Mathis Rocton**: The Parameterized Complexity Landscape of the Unsplittable Flow Problem	**Sebastian Wiederrecht**: Approximating Branchwidth on Parametric Extensions of Planarity
12:20			
12:30	lunch	lunch	lunch
14:00	*chair: J. Kratochvíl* **Amadeus Reinald**: Oriented Trees in $O(k\sqrt{k})$ -chromatic Digraphs, a Subquadratic Bound for Burr's Conjecture	*chair: C. Paul* **Van Bang Le**: On Polynomial Kernelization for Stable Cutset	
14:20	**Nathan Claudet**: Covering a Graph with Minimal Local Sets	**Kutty Malu V K**: Revisiting Path Contraction and Cycle Contraction	
14:40	**Eda Kaja**: Classification of Finite Directed Vertex-colored Ultrahomogeneous Graphs	**Felicia Lucke**: Finding d-Cuts in Graphs of Bounded Diameter, Graphs of Bounded Radius and H-Free Graphs	
15:00	**Kacper Kluk**: Graph Reconstruction with Connectivity Queries	**Jelle Oostveen**: The Complexity of Diameter on H-free graphs	
15:20	coffee break		
15:40		**Conference trip to Bled** meet in front of the hotel 15:30 bus departure 15:40 **and Conference dinner** at Restaurant Kunstelj, Radovljica bus departure from Bled 19:00	
15:50	*chair: M. Hatzel* **Nacim Oijid**: Fast Winning Strategies for the Attacker in Eternal Domination		
16:10	**Satyabrata Jana**: Roman Cycle Hitting Set		
16:30	break		
16:40	**Best Paper and Best Student Paper ceremony**		
16:50	*chair: M. Milanič* **Lijo M. Jose**: Face-hitting Dominating Sets in Planar Graphs		
17:10	**Ryan Keeney**: Degreewidth on Semi-Complete Digraphs		
17:30	**Jan Soukup**: Many Views of Planar Point Sets		
17:50	**Group photo**		
18:30	dinner		
20:00	**Business meeting**		