The role of symmetry in cage constructions

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A k-regular graph of girth g is said to be a cage if its order n(k,g) is the smallest among all k-regular graphs of girth g. The Cage Problem is the problem of finding cages and the corresponding values n(k,g) for all parameter pairs k and g. Cages are only known for very limited pairs of parameters, and the Cage Problem is universally accepted to be very hard.

A significant proportion of the known cages exhibit a high level of symmetry; with many being vertex-transitive or even Cayley. This observation is the motivation behind restricting the general Cage Problem to the problem of finding smallest vertex-transitive graphs of given degree and girth. While focusing on vertex-transitive graphs may not necessarily directly lead to finding new cages, understanding the structure of small vertex-transitive graphs of given degree and girth adds to our understanding of the general Cage Problem and may even produce graphs that could possibly be altered to become cages (by giving up some of their symmetry properties).

In our talk, we will discuss connections between general and vertex-transitive graphs of given degree and girth and present constructions of vertex-transitive graphs from non-symmetric graphs and vice versa.