Perfect matchings in cubic graphs: around Berge's conjecture

Martin Škoviera

In early 1970's, Claude Berge made a fascinating conjecture that every bridgeless cubic graph can be expressed as a union of at most five perfect matchings. After more than 50 years this conjecture remains widely open. One of the difficulties stems from the fact that cubic graphs that require five perfect matchings to cover their edges appear to be extremely rare and are difficult to construct. Until very recently, only two infinite classes of such graphs have been known. On the other hand, Berge's conjecture has been verified for just a few classes of cubic graphs, mostly having an explicit structure. In this talk we will report on a progress in both directions. First, we will describe a general approach to construction graphs requiring five perfect matchings by using a combination of projective geometry and flow theory. Second, we show that all bridgeless cubic graphs that are in a certain sense close to 3-edge-colourable graphs satisfy Berge's conjecture. Moreover, we characterise the extremal graphs.

The talk is based on a joint work with Ján Karabáš, Edita Máčajová, and Roman Nedela.