On the balanced upper chromatic number of finite projective planes

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The upper chromatic number of a hypergraph is the maximum number of colours in rainbow-free colouring (so each edge contains two points of the same colour). This notion was introduced by Voloshin. Papers by Bacsó, Tuza, Héger and the speaker show that for projective planes probably the best thing is to colour a double blocking set with one colour and the remaining points are rainbow. We only proved this for desarguesian planes of non-prime order under some conditions on the characteristic of the plane. Now we study a variant of the problem, where we further require that the sizes of the colour classes differ by at most 1. This way we obtain the notion of the balanced upper chromatic number, introduced by Araujo-Pardo, Kiss, and Montejano. The balanced upper chromatic number of the desarguesian projective plane PG(2,q) is determined. In this case either all or all but one colour classes have size 3 depending on q modulo 3. For general non-desarguesian projective planes we could determine the balanced upper chromatic number up to a multiplicative constant, using the probabilistic method.

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