Using regular graphs embeddings to represent bipartite graphs embeddings

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Cellular graphs embeddings are known as maps and cellular bipartite graphs embeddings are known as hypermaps. Hypermaps have been represented by several forms, two of which, the Walsh and the Vince map representations, are more popular. The Walsh map representation of a hypermap is a bipartite map where vertices are coloured black and white, being the black vertices the hypervertices ("vertices") of the hypermap and the white vertices the hyperedges ("edges") of the hypermpap. The faces of the bipartite map are the hyperfaces ("faces") of the hypermap. The Vince map representation is the dual where faces coloured black are the hypervertices, faces coloured white are the hyperedges and vertices correspond to hyperfaces of the hypermap. Maps correspond to subgroups of the free product $\Gamma = V_4 * C_2 = \langle R_0, R_2\rangle * \langle R_1\rangle$, while hypermaps correspond to subgroups of the "elementary" free product $\Delta = C_2 * C_2 * C_2$. The Walsh and the Vince map representations correspond to normal subgroups of index two of Γ that decompose as "elementary" free product $C_2 * C_2 * C_2$. However, Γ has three normal subgroups that decompose as "elementary" free product $C_2 * C_2 * C_2$, the third of these subgroups giving the less known Petrie-path map representation. By relaxing the "elementary" free product $C_2 * C_2 * C_2$ condition to just free product of rank 3, and under the extra condition "words of smaller length" on the generators, we get what we called "thin" map representations. The number of thin map representations of hypermaps increases to 15 (up to a restrictedly dual).