A new algebraic tool in combinatorics: Ring-semiring polynomial complexes and their applications

Luis Martínez (joint work with Maria Asunción García)

We add a new law of composition to any polynomial ring over an integral domain that is distributive with respect to multiplication, so that the polynomials with the multiplication and this new law is a semi-ring, thus enriching the initial structure. When the operands are monic polynomials, this law is known in the literature as the composed sum. We call the resulting structure a ring-semiring polynomial complex, and we use it to give explicit formulas in closed form when one of the operands has a low degree. We also give explicit formulas in closed form for the coefficients of monomials of high degree. This new algebraic framework naturally encodes the number of repetitions of the elements of a group that are obtained by making sums and differences of the elements of two subsets of the group, providing an alternative to the use of group rings. We give some combinatorial applications to the study of difference sets.