It's true: $\mathbb{Z}_3^8$ isn't a CI-group

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A Cayley (di)graph $\Gamma = \mathrm{Cay}(G; S)$ is a (D)CI-graph if whenever $\Gamma' = \mathrm{Cay}(G; T)$ is isomorphic to Γ, there is a group automorphism α of G such that $S^\alpha = T$. The group G is a (D)CI-group if every Cayley graph on G is a (D)CI-graph. A DCI group is also a CI group, but the converse is not necessarily true.

In 2003, Muzychuk proved that elementary abelian groups of sufficiently high rank are not DCI groups, but using terminology somewhat loosely, stated that they are not Cl groups. In 2007 Spiga improved Muzychuk's bound on the rank, again only considering digraphs but using the terminology of Cl groups. In 2011 Somlai further improved the bound on the rank and used graphs for every prime other than 3, but noted that the minimum rank of a non-Cl elementary abelian 3-group remained open. This note was largely overlooked (certainly I was unaware of it until recently).

I have a new proof (based on Spiga's digraph) showing that the elementary abelian 3-group of rank 8 is not a Cl-group. Along with presenting this result, I will provide a brief overview and history of the Cl problem and a more general construction for building non-CI-graphs from non-CI-digraphs.