Girth-(bi)regular graphs and finite geometries

György Kiss
ELTE, Budapest & University of Primorska, Koper

(joint work with Š. Miklavič and T. Szőnyi)

Let Γ denote a simple, connected, finite graph. For an edge e of Γ let $n(e)$ denote the number of girth cycles containing e. For a vertex v of Γ let $\{e_1, e_2, \ldots, e_k\}$ be the set of edges incident to v ordered such that $n(e_1) \leq n(e_2) \leq \ldots \leq n(e_k)$. Then $(n(e_1), n(e_2), \ldots, n(e_k))$ is called the *signature* of v. The graph Γ is said to be *girth-(bi)regular* if (it is bipartite, and) all of its vertices (belonging to the same bipartition) have the same signature.

We show that girth-(bi)regular graphs are related to (biregular) cages, finite projective and affine spaces and generalized polygons. We also present results in the spirit of stability theorems: we give upper bounds on $n(e_k) \leq M$ and show that in the case when $n(e_k) = M - \epsilon$ for some non-negative integer ϵ, then $\epsilon = 0$.

References

[1] Gy. Kiss, Š. Miklavič, T. Szőnyi, *A stability result for girth-regular graphs with even girth*, J. Graph Theory, **100** (2022), 163–181.

[2] Gy. Kiss, Š. Miklavič, T. Szőnyi, *On girth-biregular graphs*, Ars Math. Contemp., **23** (2023), #4.01.