

On the hierarchical product of graphs

Wilfried Imrich

Department Mathematics and Information Technology
Montanuniversität Leoben, Austria

The hierarchical product of graphs was introduced in [2] and the generalized hierarchical product in [3]. Both have untypical properties. They are non-commutative, but associative under certain conditions. For the associative case unique prime factorization with respect to both products was claimed in [1] for finite connected graphs.

We provide counterexamples to unique prime factorization in the case of the generalized hierarchical product and show that unique prime factorization holds for the hierarchical product, both in the associative and the non-associative case. If one drops the conditions for associativity the factorizations may become finer.

We also extend the result to compact graphs, that is, to infinite graphs without rays. For this part we invoke results from [4] about the rank of rayless graphs.

Our methods allow an exact description of the automorphism group of the hierarchical product.

On the way we also derive a straightforward linear algorithm for the prime factorization of finite trees with respect to the hierarchical product.

References

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