11th PhD Summer School in

DISCRETE MATHEMATICS

Book of Abstracts

Welcome

Dear Colleague!

What was started in 2010 as an informal research collaboration has now grown into a colorful series of international workshops and summer schools. We are glad to see many participants returning and several new ones joining the creative atmosphere of this event, which we will try to keep as relaxed and uplifting as in previous years. The organization of the meeting comes as a combined effort of the Faculty of Mathematics, Natural Sciences and Information Technologies (UP FAMNIT), the Andrej Marušič Institute (UP IAM), two members of the University of Primorska, and the Slovenian Discrete and Applied Mathematics Society, and is in line with our goal to create an international research center in algebraic combinatorics in this part of the world.

We wish you a pleasant and mathematically fruitful week at Koper.

Scientific Committee *(Ademir Hujdurović, Klavdija Kutnar, Aleksander Malnič, Dragan Marušič, Štefko Miklavič, Primož Šparl)*

General Information

11th PhD Summer School in Discrete Mathematics

UP FAMNIT, Koper, Slovenia, June 25 – July 1, 2023.

Organized by

UP FAMNIT *(University of Primorska, Faculty of Mathematics, Natural Sciences and Information Technologies);*
UP IAM *(University of Primorska, Andrej Marušič Institute);*
Slovenian Discrete and Applied Mathematics Society.

In Collaboration with

IMFM - Institute of Mathematics Physics and Mechanics;
Centre for Discrete Mathematics, UL PeF *(University of Ljubljana, Faculty of Education)*.

PhD Summer School in Discrete Mathematics Minicourses:
An introduction to eigenvalues and the ratio bound with a focus on examples
Karen Meagher, *University of Regina, Canada*
Wreath products and double coset graphs
Ted Dobson, *University of Primorska, Slovenia*

Scientific Committee:
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MINICOURSE DESCRIPTIONS

An introduction to eigenvalues and the ratio bound with a focus on examples

Karen Meagher
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This course will be an introduction to Algebraic Graph Theory. We will discuss automorphism groups of graphs, cliques and cocliques. We will define eigenvalues of graphs and see several examples, including Cayley graphs. We will show different techniques to find eigenvalues, including equitable partitions and quotient graphs. We will build up to the proof of the Delsarte-Hoffman Ratio bound and finally apply it to different graphs.

Wreath products and double coset graphs

Ted Dobson
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A graph Γ is vertex-transitive if, for any pair of vertices, there is an automorphism of Γ that maps one vertex to the other. In 1964 Sabidussi showed that every vertex-transitive graph is isomorphic to a double coset graph. So we may view the study of vertex-transitive graphs as the study of double coset graphs. At the same time, Sabidussi also explored the relationship between double coset graphs and Cayley graphs.

We will, in a self contained way, give the exact relationship between double coset graphs and Cayley graphs, which is done through the wreath product. We will also discuss consequences of this relationship, especially with regards to symmetry.

INVITED SPEAKERS

Some helpful things I have learnt over my career

Marston Conder
University of Auckland, New Zealand

This will be a highly non-standard talk! As it's a PhD summer school, I thought it could be helpful to offer some advice and insights to students from my last 40+ years in academia. These things will cover choice of research topics and projects, approaches to research problems, experimentation, looking for patterns, publication, participation in conferences, work-life balance, and a few others. I hope that many of them will be valuable.

The geometry of error-correcting codes

Simeon Ball
Polytechnic University of Catalunya, Barcelona, Spain

In this talk I will consider various error-correcting codes, including linear, additive and stabiliser codes. It is well known that if one considers the set of columns of a generator matrix of a linear code, then one can consider this set as a set of points in a finite projective space. The parameters of the code then translate over to properties of the point set. In this talk I will consider the geometry of various different types of codes and codes with certain nice properties, like Hermitian self-orthogonality. I will also discuss recent results on additive MDS codes.

Edge-transitive Cayley graphs and graph quotients

Cheryl E Praeger
The University of Western Australia, Australia

The edge-transitivity of a Cayley graph is most easily recognisable if the subgroup of 'affine maps' preserving the graph structure is itself edge-transitive. These are the so-called normal edge-transitive Cayley graphs. Each of them determines a set of quotients which are themselves normal edge-transitive Cayley graphs, and which are built from a very restricted family of groups (direct products of simple groups). We address the questions: how much information about the original Cayley graph can we retrieve from this special set of quotients? Can we ever reconstruct the original Cayley graph from them: if so, then how?

Our answers to these questions involve a subgroup determined by the Cayley graph, which has similar properties to the Frattini subgroup of a finite group. I'll discuss this and give some examples. It raises many new questions about Cayley graphs.

On Jordan schemes

Misha Muzychuk
Ben-Gurion University of the Negev, Israel

In 2003 Peter Cameron introduced the concept of a Jordan scheme and asked whether there exist Jordan schemes which are not symmetrizations of coherent configurations (proper Jordan schemes).

In my talk I'll describe several infinite series of proper Jordan schemes and present first developments in the theory of Jordan schemes - a new class of algebraic-combinatorial objects.

(k,t)-regular graphs

Gabriel Verret
University of Auckland, New Zealand

A graph is called (k,t)-regular if it is k-regular and the induced subgraph on the neighbourhood of every vertex is t-regular. We are interested in the following question: For which pairs (k,t) does there exist a (k,t)-regular graph? This is a very simple yet interesting question about which little was known. I will discuss previous knowledge as well as some new results obtained with Marston Conder and Jeroen Schillewaert.

On mixed dihedral groups and 2-arc-transitive normal covers of $K_{2^n,2^n}$

Jinxin Zhou
Beijing Jiaotong University, China

In this talk, I will introduce the notation of a mixed dihedral group, which is a group H with two disjoint subgroups X and Y, each elementary abelian of order 2^n, such that H is generated by $X \cup Y$, and $H/H' \cong X \times Y$. We will give a graph theoretic characterization of this family of groups, and this is then used to investigate the 2-arc-transitive normal covers of the 'basic' graph $K_{2^n,2^n}$.

STUDENT TALKS

Edge domination in incidence graphs

Sam Adriaensen
Vrije Universiteit Brussel, Elsene, Belgium

The *edge domination number* $\gamma_e(G)$ of a graph G is the size of the smallest subset S of its edges, such that any edge in G intersects some edge of S. In this talk, we will discuss the edge domination number of incidence graphs of some nice incidence structures. In particular, the following result is central.

Theorem 1 [1]. *Let G be the incidence graph of a symmetric $2-(v,k,\lambda)$ design D. Then $v-\gamma_e(G)$ equals the largest number α such that D contains a set X of α points and a set Y of α blocks, with no point of X incident with a block of Y.*

This leads us to explore upper and lower bounds on α in different incidence structures. This is joint work with Sam Mattheus and Sam Spiro.

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The Orbital Diameter of Primitive Permutation Groups

Kamilla Rekvényi
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Let G be a group acting transitively on a finite set Ω. Then G acts on $\Omega\times\Omega$ componentwise. Define the orbitals to be the orbits of G on $\Omega\times\Omega$. The diagonal orbital is the orbital of the form $\Delta=\{(\alpha,\alpha)|\alpha\in\Omega\}$. The others are called non-diagonal orbitals. Let Γ be a non-diagonal orbital. Define an orbital graph to be the non-directed graph with vertex set Ω and edge set $(\alpha,\beta)\in\Gamma$ with $\alpha,\beta\in\Omega$. If the action of G on Ω is primitive, then all non-diagonal orbital graphs are connected. The orbital diameter of a primitive permutation group is the supremum of the diameters of its non-diagonal orbital graphs.

There has been a lot of interest in finding bounds on the orbital diameter of primitive permutation groups. In my talk I will outline some important background information and the progress made towards finding explicit bounds on the orbital diameter. In particular, I will discuss some results on the orbital diameter of the groups of simple diagonal type and their connection to the covering number of finite simple groups [1]. I will also discuss some results for affine groups, which provides a nice connection to the representation theory of quasisimple groups.

Acknowledgments. These results are part of my PhD research supervised by Professor Martin Liebeck. The work has been supported by EPSRC Grant EP/R513052/1.

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The Suzuki and Ree groups cannot act primitively on the points of a finite generalised quadrangle

Vishnuram Arumugam
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Incidence geometry is the study of geometric structures involving a collection of points and lines along with a relation (called incidence) which tells us whether a point lies on a line. A generalised polygon is a type of point-line incidence structure that was introduced by Jacques Tits in 1959 to study the groups of Lie type as the symmetries of geometric objects. Since then, these objects have been studied extensively in the areas of group theory and finite geometry. The classification of these objects started from Weiss and Tits and many results about the existence (and non-existence) of generalised polygons under various symmetry conditions (point primitivity, flag transitivity and so on) since then. My aim is to show that the Suzuki and the Ree groups cannot act primitively on the points of a finite generalised quadrangle (which is a generalised 4-gon).

This work is supervised by Michael Giudici and John Bamberg.

Acknowledgments. The work has been supported by the Australian Government Research Training Program.

Hoffman-Singleton graph and ovals in the projective plane of order 5

Dávid Wilsch
Comenius University, Bratislava, Slovakia

The Hoffman-Singleton graph is the unique Moore graph with degree 7 and diameter 2. There is a long-standing open problem surrounding this graph. Can 7 of its copies be packed into the complete graph K_{50} such that they are edge-disjoint? In 2003, Šiagiová and Meszka [1] used methods from topological graph theory to construct a set of five edge-disjoint copies of the Hoffman-Singleton graph in K_{50} which share a common group of automorphisms of order 25.

We completely classify all possible edge-disjoint quintuples of Hoffman-Singleton graphs that share such an automorphism group and show their correspondence to special sets of ovals in projective plane of order 5.

This is joint work with my supervisor Martin Mačaj.

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Some non-existence results on m-ovoids in classical polar spaces

Valentino Smaldore
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In this talk we develop non-existence results for m-ovoids in the classical polar spaces $Q^-(2r+1,q)$, $W(2r-1,q)$ and $H(2r,q^2)$ for $r>2$. In [2] a lower bound on m for the existence of m-ovoids for the polar spaces $Q^-(2r+1,q)$, $W(2r-1,q)$ and $H(2r,q^2)$, $r>2$, is found by using the connection between m-ovoids, two-character sets, and strongly regular graphs. In [1] an improvement for the particular case $H(4,q^2)$ is obtained by exploiting the algebraic structure of the collinearity graph, and using the characterization of an m-ovoid as an intruiging set. Here, we use an approach based on geometrical and combinatorial arguments, inspired by the results from [4], to improve the bounds from [2]. The main result is contained in the following Theorem.

Theorem 1. *Let $q>2$ and $r\geq 3$. Let $\mathscr{P}'_{r,e}$ be one of the polar spaces $Q^-(2r+1,q)$, $W(2r-1,q)$ and $H(2r,q^2)$, $e=\{2,1,\frac{3}{2}\}$ respectively. Suppose that $\mathcal{O}$ is an m-ovoid in $\mathscr{P}'_{r,e}$, with (a) $r\geq 4$, or, (b) $e\in\{1,\frac{3}{2}\}$ and $(r,q,e)\neq(3,3,1)$. Then*

$$m \geq \frac{-r(1+\frac{2}{q^{r-e-1}})+\sqrt{r^2(1+\frac{2}{q^{r-1}})^2+4(q-2)(r-1)(q^{e+1}\frac{q^{r-2}-1}{q-1}+q^e+1)}}{2(q-1)}.$$

This bound asymptotically converges to

$$m \geq \frac{-r+\sqrt{r^2+4(r-1)(q-2)q^{r+e-2}}}{2(q-1)}.$$

We can adapt the approximation in the proof of the previous theorem slightly to handle the case $r=3$, $e=2$.

Theorem 2. *Suppose that $\mathcal{O}$ is an m-ovoid in $Q^-(7,q)$, for $q>2$, then*

$$m \geq \frac{-9+\sqrt{9(1+\frac{2}{q^2})^2+8(q-\frac{7}{3})(q^3+q^2+1)}}{2(q-1)}.$$

This is joint work with Jan De Beule and Jonathan Mannaert.

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Online graph exploration of minor-free graphs

Julia Baligacs
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The online graph exploration problem revolves around the fundamental question of how to explore an unknown environment. In its most basic setting, a single agent has to visit every vertex of an undirected weighted graph, subject to minimizing the total walked distance. Importantly, the agent only learns edges when visiting one of their endpoints.

In this talk, I will present a constant-competitive algorithm for the exploration problem on minor-free graphs.

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Classification of non-solvable groups whose power graph is a cograph

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A recent, active branch of research in algebraic graph theory studies constructions of graphs whose vertex set is a group G and whose edges reflect the structure of G in some way. An important example of such a graph is the power graph of a group G. Its vertices are the elements of G and there is an edge between distinct vertices x and y of G if and only if x is a power of y or y is a power of x.

We are interested in groups whose power graph is a cograph, i.e. it does not contain an induced subgraph isomorphic to a path of length four. We call such groups power-cograph groups. The problem of classifying power-cograph groups was posed by Cameron, Manna and Mehatari [1]. They solved this problem for nilpotent groups and provided a classification in the case of finite simple groups relative to number theoretic problems [2]. In this talk, I will present our classification of non-solvable power-cograph groups relative to the same number theoretic problems. Additionally, our techniques allow us to precisely describe the structure of solvable power-cograph groups.

Acknowledgments. The research leading to these results has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (EngageS: grant agreement No. 820148) and from the German Research Foundation DFG (SFB-TRR 195 “Symbolic Tools in Mathematics and their Application").

This is joint work with Jendrik Brachter.

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Some conditions implying stability of graphs

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University of Auckland, New Zealand

For a connected, non-bipartite graph X, the problem of understanding the automorphism group of the direct product $X \times Y$, with Y bipartite, often reduces to the special case $Y = K_2$. A graph X is said to be stable if $Aut(X \times K_2)$ equals $Aut(X) \times Aut(K_2)$. We discuss sufficient conditions for the stability of two families of graphs: graphs with every edge lying on a triangle and triangle-free graphs.

This is joint work with Ademir Hujdurović.

Graph-restrictive and exponent-restrictive actions

Marco Barbieri

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We consider the pairs (Γ, G), where Γ is a connected graph, and $G \leq \mathrm{Aut}(\Gamma)$ is a subgroup of automorphism which is transitive on the vertices of Γ. We say that (Γ, G) is *locally-L* if, for any vertex $\alpha \in V\Gamma$, the permutation group induced by the action of the vertex-stabiliser G_α on the neighbourhood of α is isomorphic to L. Following [1], a permutation group L is *graph-restrictive* if there exists a constant $\mathbf{c}(L)$ such that,

$$\text{for any locally-}L\text{ pair } (\Gamma, G), \quad |G_\alpha| \leq \mathbf{c}(L),$$

and *exponent-restrictive* if there exists a constant $\mathbf{e}(L)$ such that,

$$\text{for any locally-}L\text{ pair } (\Gamma, G), \quad \exp(G_\alpha) \leq \mathbf{e}(L).$$

The study of graph-restictive actions can be traced back to a celebrated theorem of W. Tutte, which states that $\mathbf{c}(L) \leq 48$ for any pair (Γ, G) with Γ cubic and L transitive (see [2]). The main conjecture of this field was posed by R. Weiss in [3], and it is still unsettled.

Weiss Conjecture. Any primitive group is graph-restrictive.

We observe that, if L is graph-restrictive, then the number of generators of G and the exponent of G_α are bounded from above by a function of the valency of the graph Γ. On the other hand, there are infinite families (Γ_n, G_n) whose common local action L is not primitive and where the number of generators of G_n cannot be bounded by a function of the valency of Γ_n. An analogous result is not known for the exponent: this is what prompted the definition of exponent-restrictive actions and the following question.

Question. Is any arbitrary group exponent-restrictive?

This is joint work with Valentina Grazian, Primož Potočnik and Pablo Spiga.

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Binomial Cayley Graphs and Applications to Dynamics on Finite Spaces

Francesco Viganò

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Binomial Cayley graphs are obtained by considering the binomial coefficient of the weight function of a given Cayley graph and a natural number. We introduce these objects and study two families: one associated with symmetric groups and the other with powers of cyclic groups. We determine various combinatorial properties of these graphs through the spectral analysis of their adjacency matrices. In the case of symmetric groups, we establish a relation between the multiplicity of the null eigenvalue and longest increasing sub-sequences of permutations by means of the RSK correspondence. Finally, we consider dynamical arrangements of finitely many elements in finite spaces, referred to as *particle-box systems*. We apply the results obtained on binomial Cayley graphs in order to describe their degeneracy.

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This is joint work with Bernat Bassols-Cornudella.

References

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Consistent cycles in arc transitive graphs

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For a graph Γ and a group $G \le \mathrm{Aut}(\Gamma)$, we say that a cycle is G-consistent, if there exists an automorphism $g \in G$, that acts on it as a 1-step rotation. The study of consistent cycles began in 1971 when J. H. Conway proved the following theorem, which was first published in [1].

Theorem 1. *Let* Γ *be a graph, and let* $G \leq \mathrm{Aut}(\Gamma)$ *act arc-transitively on* Γ. *Then the number of orbits of non-trivial* G*-consistent cycles under the action of* G *is equal to the valence of* Γ *minus one.*

For any two orbits $\mathcal{O}_1$ and $\mathcal{O}_2$ of G-consistent cycles under the action of G, we define their overlap $\mathrm{m}(\mathcal{O}_1, \mathcal{O}_2)$ as the maximal number of consecutive vertices any two cycles from those orbits have in common.

A tree of consistent cycles for a graph Γ and a group $G \leq \mathrm{Aut}(\Gamma)$ that acts arc-transitively on Γ, is a rooted directed tree T, such that the leaves of T are in bijection with orbits of G-consistent cycles, and that for every two paths from the root to a leaf in T, it holds that if the leaves represent orbits $\mathcal{O}_1$ and $\mathcal{O}_2$, then the number of vertices those two paths have in common is equal to $\mathrm{m}(\mathcal{O}_1, \mathcal{O}_2)$.

Question. Which trees can be realized as a tree of consistent cycles for some graph Γ and some group $G \leq \mathrm{Aut}(\Gamma)$?

We will consider this question for 4-valent graphs using techniques from [2].

This is joint work with Primož Potočnik.

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Vertex-primitive s-arc-transitive digraphs of almost simple groups

Lei Chen

The University of Western Australia

The property of s-arc-transitivity has been well studied for many years. Weiss proved that finite undirected graphs that are not cycles can be at most 7-arc-transitive. On the other hand, Praeger showed that for each s there are infinitely many finite s-arc-transitive digraphs that are not $(s+1)$-arc-transitive. However, G-vertex-primitive (G,s)-arc-transitive digraphs for large s seem rare. Thus we are interested in finding an upper bound for such s. In 2018, Giudici and Xia showed that it is sufficient to determine s when G is almost simple. We will show that $s \leq 1$ when G is almost simple with socle $\mathrm{Sz}(2^{2n+1})$ or $\mathrm{G}_2(3^{2n+1})$ for $n \geq 1$.

The algebraic connectivity of token graphs of a cycle

Mónica Reyes
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We study the algebraic connectivity (the smallest non-zero Laplacian eigenvalue) of token graphs. The k-token graph, $F_k(G)$, is the graph whose vertices are the k-subsets of the n vertices of G, two of which being adjacent whenever their symmetric difference is a pair of adjacent vertices in G.

The naming 'token graph' comes from an observation in Fabila-Monroy, Flores-Peñaloza, Huemer, Hurtado, Urrutia, and Wood [4], that vertices of $F_k(G)$ correspond to configurations of k indistinguishable tokens placed at distinct vertices of G, where two configurations are adjacent whenever one configuration can be reached from the other by moving one token along an edge from its current position to an unoccupied vertex. The k-token graphs are also called symmetric k-th power of graphs in Audenaert, Godsil, Royle, and Rudolph [2], and k-tuple vertex graphs in Alavi, Lick, and Liu [1]. Note that if $k = 1$, then $F_1(G) \cong G$; and if G is the complete graph K_n, then $F_k(K_n) \cong J(n,k)$, where $J(n,k)$ denotes the Johnson graph [4] which is distance-transitive (and, hence, distance-regular). Moreover, if G is bipartite, so it is $F_k(G)$ for any $k = 1,\ldots,|V|-1$.

Recently, it was conjectured by Dalfó, Duque, Fabila-Monroy, Fiol, Huemer, Trujillo-Negrete, and Zaragoza Martínez [3] that the algebraic connectivity of $F_k(G)$ equals the algebraic connectivity of G. This conjecture has already been proven when G is a complete graph, a complete bipartite graph or a tree, among other special cases in [3] and [5].

In this talk, we present results that relate the algebraic connectivities of a token graph and the original graph after removing a vertex.

Theorem 1. *Let G be a graph with a vertex i such that $\alpha(G) \geq \alpha(G \setminus i)$. Then, $\alpha(F_k(G)) \geq \alpha(F_k(G \setminus i))$.* **Theorem 2.** *Let $G = (V,E)$ be a graph on $n > 3$ vertices. If $\min_{i\in V} \frac{\alpha(G\setminus i)}{\alpha(G)} \geq \frac{1}{2}$, then $\alpha(F_2(G)) = \alpha(G)$.* These theorems allow us to prove the conjecture for 2-token graphs of some infinite families as we show in the following corollaries.

Corollary 3. *Let O_r be the odd graph of degree r. Then, $\alpha(F_2(O_r)) = \alpha(O_r)$.*

Corollary 4. *Let $G = K_{n_1,n_2,\ldots,n_r} \neq K_r$ be the multipartite complete graph on $n = \sum_{i=1}^r n_i$ vertices with $n_1 \leq n_2 \leq \cdots \leq n_r$, and $r > 2$. Then, $\alpha(F_2(G)) = \alpha(G)$.*

In the case of cycles, we present a new efficient method to compute the whole spectrum of $F_2(C_n)$ for any n by using the theory of lift graphs and a new method called over-lifts. These results allow us to prove the conjecture in the following results, regarding the cycle graphs.

Theorem 5. *The algebraic connectivity of the 2-token graph $F_2(C_n)$ equals the algebraic connectivity of the cycle C_n.*

Corollary 6. *The algebraic connectivity of the 2-token graph $F_2(G)$ of a unicyclic graph G equals the algebraic connectivity of G.*

This is joint work with Cristina Dalfó, Miquel Ángel Fiol and Arnau Messegué.

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The second largest eigenvalue of normal Cayley graphs on symmetric groups generated by cycles

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Aldous' Spectral Gap Conjecture states that the second largest eigenvalue of each connected Cayley graph on the symmetric group S_n with respect to a set of transpositions is attained by the standard representation of S_n. This celebrated conjecture, which was proposed in 1992 and completely proved in 2010, has inspired much interest in determining the second largest eigenvalue of Cayley graphs on S_n. In this talk, we focus on the normal Cayley graphs Cay$(S_n, C(n,I))$ on the symmetric group S_n, where $I \subseteq \{2,3,\ldots,n\}$ and $C(n,I)$ is the set of all cycles in S_n with length in I. We prove that the strictly second largest eigenvalue of Cay$(S_n, C(n,I))$ can only be achieved by at most four irreducible representations of S_n, and we determine further the multiplicity of this eigenvalue in several special cases. As a corollary, in the case when I contains neither $n-1$ nor n we know exactly when Cay$(S_n, C(n,I))$ has the Aldous property, namely the strictly second largest eigenvalue is attained by the standard representation of S_n, and we obtain that Cay$(S_n, C(n,I))$ does not have the Aldous property whenever $n \in I$. As another corollary of our main results, we prove a recent conjecture on the second largest eigenvalue of Cay$(S_n, C(n,\{k\}))$ where $2 \le k \le n-2$.

This is joint work with Binzhou Xia and Sanming Zhou.

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Constructing cospectral hypergraphs

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Spectral hypergraph theory mainly concerns using hypergraph spectra to obtain structural information about the given hypergraph. This field has attracted a lot of attention over the last years. The spectrum of a hypergraph can be defined in different ways. In this talk, we will focus on the spectrum of two well-known hypergraph representations: adjacency tensors and integer matrices with entries defined by the number of edges that two vertices share. Two hypergraphs are cospectral if they share the same spectrum with respect to a certain representation. By studying cospectral hypergraphs, we aim to understand which hypergraph properties cannot be deduced from their spectra. In this talk, we will show new methods for constructing cospectral uniform hypergraphs.

This is joint work with Aida Abiad.

On the zero forcing number of some graph classes in the Johnson, Grassmann and Hamming association scheme

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Zero forcing is a propagation process on a graph where the vertices are initially partitioned into two sets of black and white vertices. A white vertex is colored black (*forced*) if it is the unique white neighbor of a black vertex. The minimum number of initial black vertices needed to force all vertices of a graph G is called the *zero forcing number*. Zero forcing was initially introduced as an upper bound for the minimum rank, but has many applications in other fields of research, such as the investigation of quantum networks, influence in social networks and power dominating sets. In general, computing the zero forcing number is an NP-hard problem, and the exact value is known for only a few specific graph classes.

In this talk, we study the zero forcing number of some generalized Johnson and generalized Grassmann graphs, extending previous results for Johnson graphs on 2-sets and Kneser graphs. As a corollary, we obtain the zero forcing number of Johnson graphs, which was previously unknown. In addition, we determine the zero forcing number of Hamming graphs.

This is joint work with Aida Abiad, Robin Simoens.

Characterizing Clique Convergence for Locally Cyclic Graphs of Minimum Degree $\delta \geq 6$

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We prove that the clique graph operator k is divergent on a (not necessarily finite) locally cyclic graph G (i. e. $N_G(v)$ is a circle for every vertex v) with minimum degree $\delta(G) \geq 6$ if and only if the universal cover of G contains arbitrarily large triangular-shaped subgraphs. For finite G, this is equivalent to G being 6-regular.

The clique graph G of a graph G has the maximal complete subgraphs of G as vertices and its edges are given by non-empty intersections. The $(n+1)$-st iterated clique graph is inductively defined as the clique graph of the n-th iterated clique graph. If all iterated clique graphs of G are pairwise non-isomorphic, the graph G is called k-divergent; otherwise, it is k-convergent.

Locally cyclic graphs with $\delta \geq 6$ which induce simply connected simplicial surfaces are isomorphic to their universal covers. On this graph class, we prove our claim by explicit construction of the iterated clique graphs. After that, we show that locally cyclic graphs with $\delta \geq 6$ are k-convergent if and only if their universal covers are k-convergent. This way, we can drop the condition of simple connectivity.

This is joint work with Markus Baumeister and Martin Winter.

The distance function on Coxeter like graphs

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Let $S_n(\mathbb{F}_2)$ be the set of all $n \times n$ symmetric matrices with coefficients from the binary field $\mathbb{F}_2 = \{0, 1\}$, and let $SGL_n(\mathbb{F}_2)$ be the subset of all invertible matrices. Let $\tilde{\Gamma}_n$ be the graph with the vertex set $S_n(\mathbb{F}_2)$, where two matrices $A, B \in S_n(\mathbb{F}_2)$ form an edge if and only if $\operatorname{rank}(A - B) = 1$. Let Γ_n be the subgraph in $\tilde{\Gamma}_n$, which is induced by the set $SGL_n(\mathbb{F}_2)$. If $n = 3$, Γ_n is the Coxeter graph. It is well-known that is a distance function on $\tilde{\Gamma}_n$ is given by

$$d(A,B) = \begin{cases} \operatorname{rank}(A-B), & \text{if } A-B \text{ is nonalternate or zero,} \\ \operatorname{rank}(A-B)+1, & \text{if } A-B \text{ is alternate and nonzero.} \end{cases}$$

Even the Coxeter graph shows that the distance in Γ_n must be different. The main goal is to describe the distance function on this graph.

Joint work with Marko Orel.

Strict Erdős-Ko-Rado for simplicial complexes

Denys Bulavka
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What is the largest cardinality of a family of (pairwise) intersecting sets? A now-classic result of Erdős, Ko, and Rado answers this question if the sets all have the same number of elements, and are otherwise unrestricted.

Theorem 1. (Erdős, Ko, and Rado [2]) *Let $r \leq n/2$. If $\mathscr{F}$ is a family of intersecting subsets of $[n]$, each with r elements, then $|\mathscr{F}| \leq \binom{n-1}{r-1}$. Moreover, if $r < n/2$ then equality holds if and only if all the members of $\mathscr{F}$ have common intersection.*

Holroyd and Talbot [3], and Borg [1] conjectured that the above theorem extends to simplicial complexes as follows.

Conjecture 2. *Let K be a simplicial complex with minimal facet cardinality d and let $r \leq d/2$. If $\mathscr{F} \subseteq K$ is an intersecting family of r-faces, then $|\mathscr{F}| \leq \max_{v \in V} f_{r-1}(\operatorname{lk}(v, K))$. Moreover, if $r < d/2$ then equality holds if and only if all the members of $\mathscr{F}$ have common intersection.*

The first part of the conjecture has been shown to holds for sequentially Cohen-Macaulay near-cones [4]. We establish the second part of the conjecture for the same type of simplicial complexes.

Theorem 1. *Let K be a sequentially Cohen-Macaulay near-cone with minimal facet cardinality d and let $r < d/2$. If $\mathscr{F} \subseteq K$ is intersecting family of r-faces such that $\bigcap_{F\in\mathscr{F}} F = \emptyset$ then* $|\mathscr{F}| < \max_{v\in V} f_{r-1}(\mathrm{lk}(v,K))$.

As a corrolary we obtain that the independence complex of a chordal graph with an isolated vertex satisfy the conjecture. The novelty of our techniques is to combine algebraic and combinatorial shifting operations.

This is joint work with Russ Woodroofe.

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Convex-geometric k-planar graphs are convex-geometric $(k+1)$-quasiplanar

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A *topological* graph is a drawing of a graph in the plane with vertices as points and edges as simple curves between the vertices. We say that a topological graph is *simple* if each pair of edges crosses at most once. A particular class of topological graphs are *geometric graphs*, in which edges are drawn as straight-line segments between the vertices. An even more restricted subclass of geometric graphs are *convex-geometric* graphs, where vertices are drawn along the circle.

A topological graph is said to be k-*planar* if every edge is crossed by at most k other edges in the graph. It is said to be k-*quasiplanar* if it contains no set of k pairwise crossing edges. In 2020, Angelini et al. [1] proved that every simple topological k-planar graph is simple $(k+1)$-quasiplanar, which was the first nontrivial result concerning the relationship between these two classes. Inspired by their techniques we prove the same result in the more restrictive case of convex-geometric drawings, in particular:

Theorem 1. *Let G be a convex-geometric k-planar graph for $k > 2$. Then G is convex-geometric $(k+1)$-quasiplanar.*

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Eulerian magnitude homology: introduction and applications to random graphs

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Magnitude homology is a fairly new algebraic topology tool defined by Richard Hepworth and Simon Willerton in [1]. It was introduced in the context of undirected simple graphs, and even in this restricted setting it has proved to be a rich invariant: for example, Asao [2] has established a close relationship between magnitude homology and path homology, exploiting this relationship to prove that every diagonal graph has trivial reduced path homology. Though a lot of questions about magnitude homology have been answered and a number of possible application have been explored up to this point, magnitude homology was never exploited for the structure analysis of a graph.

Being able to use magnitude homology to look for graph substructures seems a reasonable consequence of the definition of boundary map $\partial_{k,\ell}$. Indeed, a tuple $(x_0,\dots,x_k) \in MC_{k,\ell}$ is such that $\partial_{k,\ell}(x_0,\dots,x_k) = 0$ if for every vertex $x_i \in \{x_1,\dots,x_{k-1}\}$ it holds that $len(x_{i-1},x_{i+1}) < len(x_{i-1},x_i,x_{i+1})$. In other words, $\partial_{k,\ell}(x_0,\dots,x_k) = 0$ if every vertex of the tuple is contained in a *small enough substructure*, which suggests the presence of a meaningful relationship between the rank of magnitude homology groups of a graph and the subgraph counting problem. A major problem in exploring this relationship comes from the fact that the definition of $MC_{k,\ell}(G)$ only asks for *consecutive* vertices to be different. That is, if x_0 and x_1 are two adjacent vertices in G an acceptable tuple in $MC_{5,4}(G)$ is (x_0,x_1,x_0,x_1,x_0). Tuples of this kind inducing a path that just revisits again and again the same edge (an more in general, tuples inducing non-eulerian walks) do not provide any insight about the meaning of magnitude homology. With this motivation, we introduce the *eulerian magnitude chain* a slightly different definition of magnitude chain, considering the subgroup of $MC_{k,l}(G)$ where an edge is never required to be revisited. We then define *eulerian magnitude homology*, $EMH_{k,l}(G)$, and the investigation of the meaning of rank$(EMH_{k,l}(G))$ leads us to the following simple but promising result:

Theorem 1. *Let Z be the number of basis elements $\overline{x} \in EMC_{k,k}(G)$ such that $\partial_{k,k}(\overline{x}) = 0$. Then the number of k-cliques in G is upper bounded by $\lfloor \frac{Z}{k!} \rfloor$.*

We then reproduce the analysis performed by Matthew Khale and Elizabeth Meckes in [3] in order to analyze the behavior of $EMH_{k,k}(G)$ in case G is an Erdos-Renyi graph and random geometric graphs, obtaining the following results:

Theorem 2. *Let $G = G(n,p)$ be an Erdos-Renyi graph on n vertices and set $p = n^{-\alpha}$. The eulerian magnitude homology groups $EMH_{k,k}(G)$ vanish for $\alpha > \frac{k+1}{2k-1}$.*

Theorem 3. *Let $G = G(n,p)$ be an Erdos-Renyi graph and call $\beta_{k,k} = rank(EMH_{k,k}(G))$. Then for every p it holds that $\lim\limits_{n\to\infty} \frac{\mathbb{E}(\beta_{k,k})}{n^{k+1}p^k} = \frac{1}{(k+1)!}$.*

Theorem 4. *Let $G(n,r,A)$ be a random geometric graph and take $r = n^{-\alpha}$. The eulerian magnitude homology groups $EMH_{k,k}(G)$ vanish for $\alpha > \frac{k+1}{2k}$.*

Theorem 5. *Let $G = G(n,r,A)$ be a random geometric graph and call $\beta_{k,k} = rank(EMH_{k,k}(G(n,r)))$. Then $\lim\limits_{n\to\infty} \frac{\mathbb{E}(\beta_{k,k})}{n^{k+1}r^{2k}} = \left(\frac{\pi}{|A|}\right)^k \frac{1}{(k+1)!}$.*

This is joint work with Giulia Bernardini, Chad Giusti and Luca Manzoni.

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Hamiltonian path and cycle in graphs of bounded independence number

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A Hamiltonian path (a Hamiltonian cycle) in a graph is a path (a cycle, respectively) that traverses all of its vertices. The problems of deciding their existence in an input graph are well-known to be NP-complete, in fact, they belong to the first problems shown to be computationally hard when the theory of NP-completeness was being developed. A lot of research has been devoted to the complexity of Hamiltonian path and Hamiltonian cycle problems for special graph classes, yet only a handful of positive results are known. The complexities of both of these problems have been open even for $4K_1$-free graphs, i.e., graphs of independence number at most 3. We answer this question in the general setting of graphs of bounded independence number.

We also consider a newly introduced problem called *Hamiltonian-ℓ-Linkage* which is related to the notions of a path cover and of a linkage in a graph. This problem asks if given ℓ pairs of vertices in an input graph can be connected by disjoint paths that altogether traverse all vertices of the graph. For $\ell = 1$, Hamiltonian-1-Linkage asks for existence of a Hamiltonian path connecting a given pair of vertices. Our main result reads that for every pair of integers k and ℓ, the Hamiltonian-ℓ-Linkage problem is polynomial time solvable for graphs of independence number not exceeding k. We further complement this general polynomial time algorithm by a structural description of obstacles to Hamiltonicity in graphs of independence number at most k for small values of k.

This is joint work with Jan Kratochvil.

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Commutators in completely simple semigroups

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Nilpotency and solvability of semigroups are properties defined by generalizing the notion of the commutator from group theory. The *term condition commutator* $[\alpha,\beta]$ of congruences α,β on semigroup $\mathbf{S}$ is the least congruence δ which satisfies the *term condition*: for every $(n+1)$-ary term $t = t(x, y_1,\ldots, y_n)$, and for every $a, b \in S$, $(c_1,\ldots,c_n)$, $(d_1,\ldots,d_n) \in S^n$ such that $a\ \alpha\ b$ and $c_i\ \beta\ d_i$, $i = 1,\ldots,n$ we have that $t(a,c_1,\ldots,c_n)\ \delta\ t(a,d_1,\ldots,d_n)$ implies $t(b,c_1,\ldots,c_n)\,\delta\, t(b,d_1,\ldots,d_n)$.

A semigroup $\mathbf{S}$ is *abelian* if it satisfies the equality $[1_S,1_S] = 0_S$, where $1_S = S \times S$, and 0_S is the equality relation on S. Further on, a semigroup $\mathbf{S}$ is n*-nilpotent*, $n \in \mathbb{N}$ if $\underbrace{[1_S,[1_S,\ldots,[}_{n}1_S,1_S]\ldots]] = 0_S$. Solvability is defined similarly, using the generalized solvable series. Abelianess of semigroups has been completely described in [5] by R.J. Warne. If the semigroup $\mathbf{S}$ contains the zero element $\{0\}$, then it is n-nilpotent (solvable) if and only if $S^{n+1} = \{0\}$ ($S^m = \{0\}$ for some $m \in \mathbb{N}$), as proved in [4]. Therefore, the characterization of nilpotency and solvability remains unsolved for semigroups without the zero element.

One fundamental class of semigroups without zero are completely simple semigroups. A semigroup $\mathbf{S}$ is *completely simple* if it has no proper ideals, and it contains a primitive idempotent. A *primitive idempotent* is the element e that uniquely satisfies the equations $ex = xe = x$ on x. Completely simple semigroups are uniquely described with Rees matrix semigroups. The Rees matrix semigroup over sets I, Λ and group $\mathbf{G}$ with the structure matrix $P_{\Lambda\times I}$, $p_{\lambda i} \in G$, is the direct product $I \times G \times \Lambda$ with the multiplication defined by $(i,g,\lambda)\cdot(j,h,\mu) = (i,gp_{\lambda j}h,\mu)$. This combinatorial representation allows for an easier approach in studying their (universal) algebraic properties, and allows us to obtain the following description of the commutator in completely simple semigroups:

Theorem 1. *Let* $\mathbf{S} = \mathscr{M}[G;I,\Lambda;P]$ *be a completely simple semigroup. Let* ρ,σ *be congruences on* $\mathbf{S}$*, Then commutator* $[\rho,\sigma]$ *corresponds to the linked triple* $(0_I,[\rho_G,\sigma_G]\vee\Theta_{\rho,\sigma},0_\Lambda)$.

Here $\Theta_{\rho,\sigma}$ denotes the congruence determined by the matrix P and components of ρ and σ on the index sets I and Λ. We also prove the equivalence between the nilpotency (solvability) of a completely simple semigroup $\mathbf{S} = \mathscr{M}[G;I,\Lambda;P]$ and the group $\mathbf{G}$:

Theorem 2. *Let* $\mathbf{S} = \mathscr{M}[G;I,\Lambda;P]$ *be a completely simple semigroup, then the semigroup* $\mathbf{S}$ *is nilpotent (solvable) if and only if* $\mathbf{G}$ *is a nilpotent (solvable) group.*

This is joint work with Nebojša Mudrinski.

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Intersecting diametral balls induced by a geometric graph

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For a graph whose vertex set is a finite set of points in $\mathbb{R}^d$, consider the closed (open) balls with diameters induced by its edges. The graph is called *a (an open) Tverberg graph* if these closed (open) balls intersect. Using the idea of halving lines, we show that (*i*) for any finite set of points in the plane, there exists a Hamiltonian cycle that is a Tverberg graph; (*ii*) for any n red and n blue points in the plane, there exists a perfect red-blue matching that is a Tverberg graph. Also, we prove that (*iii*) for any even set of points in $\mathbb{R}^d$, there exists a perfect matching that is an open Tverberg graph; (*iv*) for any n red and n blue points in $\mathbb{R}^d$, there exists a perfect red-blue matching that is a Tverberg graph.

Theorem 1. *For any finite set of points in the plane, a Tverberg cycle exists.*

Theorem 2. *For any set of n red points and n blue points in the plane, a Tverberg red-blue matching exists.*

Theorem 3. *For any even set of distinct points in $\mathbb{R}^d$, an open Tverberg matching exists.*

Theorem 4. *For n red points and n blue points in $\mathbb{R}^d$, a red-blue Tverberg matching exists.*

This is joint work with A.Polyanskii and A.Vasilevskii.

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A few words about Slovenian Discrete and Applied Mathematics Society

Slovenian Discrete and Applied Mathematics Society was founded in Koper (Slovenia), on 14 December 2016. The aim of this society is to promote the mathematical sciences, with special emphasis given to discrete and applied mathematics. The Society is research-oriented, and publishes scientific literature and organises scientific meetings such as this one. In particular, it is involved in publishing *Ars Mathematica Contemporanea* and *The Art of Discrete and Applied Mathematics* . It has members, fellows and honorary members.

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With international faculty and many research links all over the world, UP FAMNIT and its research counterpart UP IAM are at the forefront of the academic development of UP. Student enrollment at UP FAMNIT has grown from approximately 100 in its first academic year (2007/08), to 929 in the academic year 2022/23.

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- AC^2 – Algebraic Combinatorics on the Adriatic Coast, Koper, 2003, 2004, 2008, 2009.
- CoCoMat – Korea - Slovenia International Conference On Combinatorial and Computational Mathematics, Koper, 2007.
- International Workshop on Symmetries of Graphs and Networks 2010, 2012, 2014.
- PhD Summer Schools in Algebraic Graph Theory 2011 and Discrete Mathematics 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2022.
- 7th Slovenian International Conference on Graph Theory, Bled, 2011.
- Computers in Scientific Discovery 6, August 2012.
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- 2015 International Conference on Graph Theory, May 2015.
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- Graphs, groups, and more: celebrating Brian Alspach's 80th and Dragan Marušič's 65th birthdays, Koper, May-June, 2018.
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Ars Mathematica Contemporanea (AMC) is an international journal, published by UP in collaboration with IMFM, the Slovenian Discrete and Applied Mathematics Society and the Slovenian Society of Mathematicians, Physicists and Astronomers.

The aim of AMC is to publish peer-reviewed high-quality articles in contemporary mathematics that arise from the discrete and concrete mathematics paradigm. It favors themes that combine at least two different fields of mathematics. In particular, papers intersecting discrete mathematics with other branches of mathematics, such as algebra, geometry, topology, theoretical computer science, and combinatorics, are most welcome.

In 2015 the Ars Mathematica Contemporanea Journal (AMC) was ranked as the best Slovene scientific journal. Its impact factor for 2015 was 0.985, which landed the journal in the top quartile for scientific journals in the field of mathematics. In 2016 and 2017, the journal was placed in the second quartile with impact factors 0.87 and 0.793, respectively. In 2021 the Ars Mathematica Contemporanea Journal (AMC) was ranked second among all Slovene scientific journal. Its impact factor for 2021 was 0.922.

This journal was launched in 2008 by Tomaž Pisanski and Dragan Marušič, and the editorial board today is led by Editors in Chief, Klavdija Kutnar, Dragan Marušič and Tomaž Pisanski.

For more information on submissions, please refer to the AMC website

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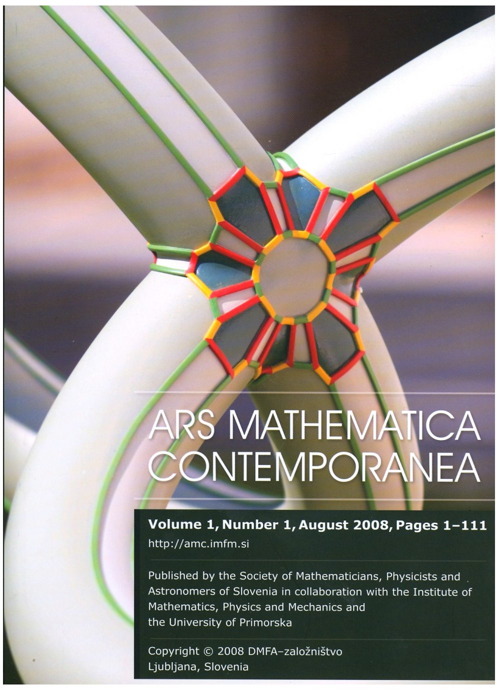

THE ART OF DISCRETE AND APPLIED MATHEMATICS

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