Graphs and Groups, Geometries and GAP (G2G2) Summer School - External Satellite Conference of 8ECM

Report of Contributions

Contribution ID: **1** Type: **not specified**

Associative Subalgebras of Majorana Algebras

A Majorana algebra is a commutative nonassociative real algebra generated by a finite set of idempotents, called Majorana axes, that satisfy some of the properties of the $2A$-axes of the Monster Griess algebra. The term was introduced by A. A. Ivanov (2009) inspired by the work of S. Sakuma and M. Miyamoto. In this talk, we are going to revisit, in the context of Majorana theory, the theorem of Mayer and Neutsch (1993) that states that any maximal associative subalgebra of the Griess algebra has an orthogonal basis of indecomposable idempotents that add up to the identity. We apply this result to determine all the maximal associative subalgebras of some low-dimensional Majorana algebras; namely, the two-generated Majorana algebras and the Majorana representations of S_4 involving $3C$-algebras.

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Track Classification: Oral presentation

Contribution ID: **4** Type: **not specified**

Multi-switches, representations of braid groups and invariants of virtual links.

The Yang-Baxter equation first appeared in theoretical physics and statistical mechanics in the works of Yang (1967) and Baxter (1972) and it has led to several interesting applications in different fields of mathematics. For example, the Yang-Baxter equation appears in topology (namely, in knot theory) and algebra since it is strongly connected with braid groups. The problem of studying set-theoretical solutions of the Yang-Baxter equation was formulated by Drinfel'd (1992).

In the talk, we will give a short introduction to the Yang-Baxter equation, and introduce a new original way of how solutions of this equation can be used for constructing representations of (virtual) braid groups and invariants of (virtual) knots and links.

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Track Classification: Oral presentation

Contribution ID: **5** Type: **not specified**

The nonexistence of distance-regular graphs with intersection array {56,42,20;1,6,28} in which some local subgraph is the 7x8-grid.

There is an infinite sequence of formally self-dual classical distance-regular graphs Γ with classical parameters $b = 2$, $\alpha = 1$, $\beta = n - 1$, $v = n^3$ $(n > 5)$.

If n is a power of 2, then there exists a distance-regular graph Γ with intersection array $\{7(n-1), 6(n-2), 4(n-4); 1, 6, 28\}$ and each distance-regular graph with these parameters is a bilinear forms graph. Moreover, all these graphs are locally grid.

We consider graphs Γ with intersection array $\{7(n-1), 6(n-2), 4(n-4); 1, 6, 28\}$, where n is not a power of 2. By the results of Metsch, if $n \geq 71$, then graphs with intersection array $\{7(n-1), 6(n-2), 4(n-4); 1, 6, 28\}$ don't exist.

If $n = 6$, then the intersection array of Γ is $\{35, 24, 8; 1, 6, 28\}$. The nonexistence of a graph with this intersection array was proved by A. Jurishich and J. Vidali. The case when $n = 7$ was ruled out by I.N. Belousov and A.A. Makhnev. The proof was based on counting of some triple intersection numbers.

In this work, we consider the case $n = 9$. In this case, there are 6 admissible spectra for the integral local subgraph. One of them is relate to the 7×8-grid.

We prove that a distance-regular graph with intersection array $\{56, 42, 20; 1, 6, 28\}$ does not exist, if some its local subgraph is the 7×8-grid.

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Track Classification: Oral presentation

Contribution ID: **6** Type: **not specified**

On the Weisfeiler-Leman dimension of some classes of graphs

The *Weisfeiler-Leman* algorithm (*WL-algorithm*) is a fundamental algorithm used as a subroutine in graph isomorphism testing. The classical WL-algorithm [3] constructs for a given graph the coherent closure of its edge set in polynomial time. For every positive integer k there is a k-dimensional version of the WL-algorithm which deals with k-tuples of vertices (see Section 4.6 [1] for details). In this terms the classical WL-algorithm is 2-dimensional.

Following Grohe (Definition 18.4.2 in [2]), we define the *WL-dimension* $dim_{WL}(\mathcal{K})$ of a class $\mathcal{K}$ of graphs to be the minimal integer m such that the m-dimensional WL-algorithm distinguishes each graph $\Gamma \in \mathcal{K}$ from all graphs not isomorphic to Γ. If $dim_{WL}(\mathcal{K}) = m$ then the isomorphism of two n-vertex graphs from $\mathcal{K}$ can be checked in time $n^{O(m)}$. The WL-dimension is bounded for many natural graph classes including trees, cographs, interval graphs, planar graphs. In the talk we discuss on the approach to estimate an upper bound of the WL-dimension based on separability property of coherent configurations and present new upper bounds for the WL-dimension of some classes of graphs.

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Track Classification: Oral presentation

Contribution ID: **7** Type: **not specified**

On the pronormality of subgroups of odd index in some direct products of finite groups

We consider only finite groups. A subgroup H of a group G is said to be *pronormal* in G if H and H^g are conjugate in $\langle H, H^g \rangle$ for each $g \in G$. Some of well-known examples of pronormal subgroups are the following: normal subgroups; maximal subgroups; Sylow subgroups; Sylow subgroups of proper normal subgroups; Hall subgroups of solvable groups. Some problems in finite group theory, combinatorics, and permutation group theory were solved in terms of the pronormality, see, for example [1, 5]. Thus, the problem of pronormality of a subgroup (in particular, of a subgroup of odd index) in a finite group is of interest. Detailed surveys of investigations on pronormality of subgroups of odd index in finite groups could be found in survey papers [2]. These surveys contain new results and some conjectures and open problems. One such open problem is to complete the classification of finite simple groups in which all the subgroups of odd index are pronormal. One more open problem involves the classification of direct products of nonabelian simple groups in which the subgroups of odd index are pronormal. A detailed motivation for these problems was provided in [4].

In this talk we concentrate on the latter problem. Note that there are examples of nonabelian simple groups G such that all the subgroups of odd index are pronormal in G, but the group $G \times G$ contains a non-pronormal subgroup of odd index (see Proposition 1 in [4]). We prove the following theorem.

Theorem. *Let $G = \prod_{i=1}^{t} G_i$, where for each $i \in \{1, ..., t\}$, $G_i \cong Sp_{n_i}(q_i)$ for odd q_i. Then the following statements are equivalent:

(1) all the subgroups of odd index are pronormal in G;

(2) for each i, all the subgroups of odd index are pronormal in G_i;

(3) for each i, if $q_i \equiv \pm 3 \pmod 8$, then n_i is either a power of 2 or is a number of the form $2^w(2^{2k}+1)$.*

Moreover, guided by the analysis in the proof of Theorem, we conclude that deciding the pronormality of a given subgroup of odd index in the direct product of simple symplectic groups over fields of odd characteristics is reducible to deciding the pronormality of some subgroup H of odd index in a subgroup of a group $\prod_{i=1}^{t} Z_i \wr Sym_{n_i}$, where $Z_i \in \{1, C_3\}$ for each i, such that H projects onto $\prod_{i=1}^{t} Sym_{n_i}$, and we have obtained a criterium of pronormality of such a subgroup in such a group. These investigations give a rise to researches on effective algorithms (to be implemented in GAP) for deciding the pronormality of a subgroup of odd index in a finite simple symplectic group, the speaker is working on these algorithms in a joint project with Stephen Glasby and Cheryl E. Praeger.

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Track Classification: Oral presentation

Contribution ID: **8** Type: **not specified**

On coincidence of Gruenberg–Kegel graphs of non-isomorphic finite groups

We use mainly standard notation and terminology (see [1]). Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of the order of G and by $\omega(G)$ the *spectrum* of G, i.e. the set of all its element orders. The set $\omega(G)$ defines the *Gruenberg–Kegel* graph (or the *prime graph*) $\Gamma(G)$ of G; in this simple graph the vertex set is $\pi(G)$, and distinct vertices p and q are adjacent if and only if $pq \in \omega(G)$.

The problem of description of the cases when Gruenberg–Kegel graphs of non-isomorphic finite groups coincide naturally arises. One of the most interesting cases of this problem is when Gruenberg–Kegel graphs of non-isomorphic finite groups are both disconnected and coincide.

Recall some known definitions. A finite group G is a *Frobenius group* if there is a non-trivial subgroup C of G such that $C \cap gCg^{-1} = \{1\}$ whenever $g \notin C$. A subgroup C is a *Frobenius complement* of G. Let $K = \{1\} \cup (G \setminus \bigcup_{g\in G} gCg^{-1})$. Then K is a normal subgroup of a Frobenius group G with a Frobenius complement C which is called the *Frobenius kernel* of G. A finite group G is a 2-Frobenius group if $G = ABC$, where A and AB are normal subgroups of G, AB and BC are Frobenius groups with kernels A and B and complements B and C, respectively. It is known that each 2-Frobenius group is solvable. The *socle* $Soc(G)$ of a finite group G is the subgroup of G generated by the set of all its non-trivial minimal normal subgroups. A finite group G is *almost simple* if $Soc(G)$ is a finite nonabelian simple group.

By the Gruenberg–Kegel Theorem, if G is a finite group with disconnected Gruenberg–Kegel graph, then either G is a Frobenius group or either G is a 2-Frobenius group or G is an extension of a nilpotent group by an almost simple group. In [2] all the cases of coincidence of Gruenberg–Kegel graphs of a finite simple group and of a Frobenius or a 2-Frobenius group were described. Moreover, we can obtain the complete list of almost simple groups whose Gruenberg–Kegel graphs coincide with Gruenberg–Kegel graphs of solvable Frobenius groups or 2-Frobenius groups directly from the main results of the papers [3] and [4].

In this talk we concentrate on a list of almost simple (but not simple) groups whose Gruenberg–Kegel graphs coincide with Gruenberg–Kegel graphs of non-solvable Frobenius groups. We prove the following theorem.

Theorem 1. *Let G be an almost simple but not simple group. If $\Gamma(G) = \Gamma(H)$, where H is a non-solvable Frobenius group, then either G is one of the groups $Aut(M_{12})$, S_7, $Aut(^2F_4(q))$, $Aut(PSU_4(2))$, $L_3(4).2_3$, $PSL_4(3).2_2$, $PSL_4(3).2_3$, $PSL_2(11).2 \cong PGL_2(11)$, $PSL_2(19).2 \cong PGL_2(19)$, $PGL_2(49).2_1 \cong PGL_2(49)$, $PSL_2(25).2_2$, $PSL_2(81).2_1$, $PSL_2(81).4_1$, $PSL_2(81).4_2$; or $Soc(G) \in \{PSL_3(q), PSU_3(q)\}$.*

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Track Classification: Oral presentation

Contribution ID: **9** Type: **not specified**

On finite non-solvable 4-primary groups without elements of order 6

We use mainly standard notation and terminology (see [1]).

Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of the order of G. If $|\pi(G)| = n$, then G is called n-primary.

The Gruenberg–Kegel graph (prime graph) $\Gamma(G)$ of G is a graph with the vertex set $\pi(G)$, in which two distinct vertices p and q are adjacent if and only if there exists an element of order pq in G.

In 2012–2013, A.S. Kondrat'ev described finite groups having the same Gruenberg-Kegel graph as the groups $Aut(J_2)$ [2] and A_{10} [3], respectively. The Gruenberg–Kegel graphs of these groups are isomorphic as abstract graphs.

We establish a more general problem: to describe finite groups whose Gruenberg-Kegel graphs are isomorphic as abstract graphs to the graph $\Gamma(A_{10})$.

As a part of the solution of this problem, we proved in [4] that if G is a finite non-solvable group and the graph $\Gamma(G)$ as abstract graph is isomorphic to the graph $\Gamma(A_{10})$, then the quotient group $G/S(G)$ (where $S(G)$ is the solvable radical of G) is almost simple, and classified all finite almost simple groups whose the Gruenberg-Kegel graphs as abstract graphs are isomorphic to subgraphs of $\Gamma(A_{10})$.

Let G be a finite non-solvable group, and the graph $\Gamma(G)$ as abstract graph is isomorphic to the graph $\Gamma(A_{10})$. Then the graph $\Gamma(G)$ has the following form (see attachment pdf), where p, q, r, and s are pairwise distinct primes.

In [5], we described such finite non-solvable groups G when 3 does not divide $|G|$.

In this work, we prove the following theorem.

Theorem. *Let G be a finite non-solvable group, $\overline{G} \cong G/S(G)$ and $\Gamma(G)$ as abstract graph is isomorphic to $\Gamma(A_{10})$. If 3 divides of $|G|$ and G has no elements of order 6, then one of the following statements holds:*

(1) $S = O_{2',2}(G)$, $O(G) = O_p(G)$, $q = 2$, $S/O(G)$ *is an elementary abelian* 2*-group,* $\overline{G} \cong L_2(2^n)$ *and one of the following statements holds:*

$(1a)$ $n = 4$, $p = 17$ *and* $\{r, s\} = \{3, 5\}$;

$(1b)$ n *is prime,* $n \geq 5$, $p = 2^n - 1$, *and* $\{r, s\} = \{3, (2^n - 1)/3\}$, *the group* $S/O(G)$ *either is trivial or as* $\overline{G}$*-module is isomorphic to a direct sum of natural* $GF(2^n)\overline{G}$*-modules;*

(2) $S = O_p(G)$, $q = 2$, $\overline{G} \cong L_2(p)$, $p \geq 31$, $p \equiv \varepsilon 5 (mod\ 12)$, $\varepsilon \in \{+, -\}$, $p - \varepsilon 1 = 2^k$, *and* $3 \in \{r, s\} = \pi((t + \varepsilon 1)/2)$;

(3) $S = O_p(G)$, $q = 3$, *and one of the following statements holds:*

$(3a)$ $\overline{G} \cong PGL_2(9)$, $p > 5$, *and* $\{r, s\} = \{2, 5\}$;

$(3b)$ $\overline{G} \cong L_2(81)$, $PGL_2(81)$ *or* $L_2(81).2_3$, $p = 41$, *and* $\{r, s\} = \{2, 5\}$;

$(3c)$ $\overline{G} \cong L_2(3^n)$ *or* $PGL_2(3^n)$, n *is an odd prime,* $p = (3^n - 1)/2$, *and* $\{r, s\} = \pi(3^n + 1)$.

Each of the statements (1)–(3) of the theorem is realized.

In the proof of Theorem, we use the classification of the finite non-solvable groups without elements of order 6 from (Theorem 2, [6]).

Acknowledgments. The work is supported by Russian Science Foundation (project 19-71-10067).

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Track Classification: Oral presentation

Contribution ID: **10** Type: **not specified**

On maximal cliques in Paley graphs of square order

In [1], Blokhuis studied maximum cliques in Paley graphs of square order $(^2)$. It was shown that a clique of size q in $(^2)$ is necessarily a quadratic line in the corresponding affine plane $(2,)$.

Let $()$ denote the reminder after division of by 4. In [2], for any odd prime power , a maximal (but not maximum) clique in $(^2)$ of size $\frac{+()}{2}$ was constructed.

In [3], for any odd prime power , a maximal clique in $(^2)$ of the same size $\frac{+()}{2}$ was constructed. This clique was shown to have a remarkable connection with eigenfunctions of $(^2)$ that have minimum cardinality of support $+1$.

In this talk, we discuss the constructions of maximal cliques from [2] and [3] and establish a correspondence between them.

Acknowledgments. Sergey Goryainov and Leonid Shalaginov are supported by RFBR according to the research project 20-51-53023.

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Track Classification: Oral presentation

Contribution ID: **11** Type: **not specified**

s-PD-sets for codes from projective planes $\mathrm{PG}(2, 2^h)$, $5 \leq h \leq 9$

In this talk we will describe a construction of 2-PD-sets of 16 elements for codes from the Desarguesian projective planes $\mathrm{PG}(2, q)$, where $q = 2^h$ and $5 \leq h \leq 9$. We will also describe a construction of 3-PD-sets of 75 elements for the code from the Desarguesian projective plane $\mathrm{PG}(2, q)$, where $q = 2^9$. These 2-PD-sets and 3-PD-sets can be used for partial permutation decoding of codes obtained from the Desarguesian projective planes.

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Track Classification: Oral presentation

Contribution ID: **12** Type: **not specified**

Type IV-II codes over $\mathbb{Z}_4$ constructed from generalized bent functions

A Type IV-II $\mathbb{Z}_4$-code is a self-dual code over $\mathbb{Z}_4$ with the property that all Euclidean weights are divisible by eight and all codewords have even Hamming weight.
The subject of this talk is a construction of Type IV-II codes over $\mathbb{Z}_4$ from generalized bent functions.

We use generalized bent functions for a construction of self-orthogonal codes over $\mathbb{Z}_4$ of length 2^m, for m odd, $m \geq 3$, and prove that for $m \geq 5$ those codes can be extended to Type IV-II $\mathbb{Z}_4$-codes. From that family of Type IV-II $\mathbb{Z}_4$-codes, we construct a family of self-dual Type II binary codes by using the Gray map.

We also consider the weight distributions of the obtained codes and the structure of the supports of the minimum weight codewords, which we use for a construction of 1-designs.
Some of the constructed 1-designs are affine resolvable 1-designs.
For the constructed 1-designs, we examine the properties of the corresponding block intersection graphs and obtain strongly regular graphs in two cases.

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Track Classification: Oral presentation

Contribution ID: **13** Type: **not specified**

LDPC codes from cubic semisymmetric graphs

A regular graph is semisymmetric if it is edge-transitive but not vertex-transitive. A cubic semisymmetric graph is a 3-regular graph which is semisymmetric. It has been proved that every semisymmetric graph is necessarily bipartite graph. In this talk we study low-density parity-check (LDPC) codes having cubic semisymmetric graphs as their Tanner graphs. We will discuss some of the properties of the constructed codes and present bounds for the code parameters: code length, dimension and minimum distance. Further, we will discuss the structure of the smallest absorbing sets of these LDPC codes and give an expression for the variance of the syndrome weight of the constructed codes. Moreover, computational and simulation results on the constructed codes will be presented.

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Track Classification: Oral presentation

Contribution ID: **14** Type: **not specified**

k-closures of finite nilpotent permutation groups

Let G be a permutation group on a finite set Ω. Denote the set of orbits of the componentwise action of G on Ω^k by $\mathrm{Orb}(G, \Omega^k)$. Wielandt [1] defined the *k-closure* of G to be the group

$$G^{(k)} = \mathrm{Aut}(\mathrm{Orb}(G, \Omega^k)) = \{g \in \mathrm{Sym}(\Omega) \mid O^g = O \ \forall O \in \mathrm{Orb}(G, \Omega^k)\}.$$

A permutation group is called *k-closed* if $G = G^{(k)}$. In this talk we discuss k-closures of nilpotent groups.

Theorem. If G is a finite nilpotent permutation group, and $k \geq 2$, then $G^{(k)}$ is the direct product of k-closures of Sylow subgroups of G.

This theorem generalizes results of [2,3] and provides a criterion of the k-closedness for finite nilpotent permutation groups.

Corollary. For $k \geq 2$, a finite nilpotent permutation group G is k-closed if and only if every Sylow subgroup of G is k-closed.

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Track Classification: Oral presentation

Contribution ID: **15** Type: **not specified**

LCD codes obtained from weakly p-self-orthogonal designs

A 1-design is weakly p-self-orthogonal if all the block intersection numbers gives the same residue modulo p. In [1], we analyze extensions of the incidence matrix, orbit matrix and submatrices of orbit matrix of a weakly p-self-orthogonal 1-design in order to construct self-orthogonal codes.

A linear codes is called LCD code if the intersection with its dual code is trivial. Matrix G generates an LCD code if and only if $\det(G \cdot G^T) \neq 0$ (see [3]).

We extend the methods of construction described in [1] in order to construct LCD codes over finite fields. We use suitable extensions of incidence matrix, orbit matrices and submatrices of orbit matrices in order to construct LCD codes over finite field. We will present examples of LCD codes constructed from weakly p-self-orthogonal designs obtained from groups using construction described in [2].

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Rigidity of v_3-configurations

A framework of a graph is rigid if no motion of the graph, preserving edge lengths, changes the distance between two vertices. Equivalently, a graph is rigid if it can only move by translation and rotation. A graph that is not rigid is flexible. Under certain conditions on the coordinates of the vertices, we can determine whether or not a planar framework of a graph is rigid by looking at the underlying graph.

A geometric v_k-configuration is a collection of v straight lines and v points in the plane such that there are k points on each line and k lines through each points. A geometric v_2-configuration is a framework of a graph, so geometric v_k-configurations generalise frameworks of graphs.

In this talk we will consider the rigidity properties of v_k-configurations. A geometric v_k-configuration is rigid if the only motions of it, preserving point-line incidences and distances between collinear points, are translation and rotation. A geometric v_k-configuration that is not rigid is flexible.

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LCD codes from two-class association schemes

Linear codes with complementary duals, shortly named LCD codes, are linear codes whose intersection with their duals is trivial. In this talk, we present a construction for LCD codes over finite fields from the adjacency matrices of two-class association schemes. These schemes consist of either strongly regular graphs (SRGs) or doubly regular tournaments (DRTs).

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The Orbital Diameter of Primitive Permutation Groups

Let G be a group acting transitively on a finite set Ω. Then G acts on $\Omega \times \Omega$ componentwise. Define the orbitals to be the orbits of G on $\Omega \times \Omega$. The diagonal orbital is the orbital of the form $\Delta = \{(\alpha, \alpha) | \alpha \in \Omega\}$. The others are called non-diagonal orbitals. Let Γ be a non-diagonal orbital. Define an orbital graph to be the non-directed graph with vertex set Ω and edge set $(\alpha, \beta) \in \Gamma$ with $\alpha, \beta \in \Omega$. If the action of G on Ω is primitive, then all non-diagonal orbital graphs are connected. The orbital diameter of a primitive permutation group is the supremum of the diameters of its non-diagonal orbital graphs.

There has been a lot of interest in finding bounds on the orbital diameter of primitive permutation groups. In my talk I will outline some important background information and the progress made towards finding specific bounds on the orbital diameter. In particular, I will discuss some results on the orbital diameter of the groups of simple diagonal type and their connection to the covering number of finite simple groups. I will also discuss some results for affine groups, which provides a nice connection to the representation theory of quasisimple groups.

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