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k-closures of finite nilpotent permutation groups

Let G be a permutation group on a finite set Ω. Denote the set of orbits of the componentwise action of G on Ω^k by $\mathrm{Orb}(G, \Omega^k)$. Wielandt [1] defined the k-*closure* of G to be the group

$$G^{(k)} = \mathrm{Aut}(\mathrm{Orb}(G, \Omega^k)) = \{g \in \mathrm{Sym}(\Omega) \mid O^g = O \; \forall O \in \mathrm{Orb}(G, \Omega^k)\}.$$

A permutation group is called k-*closed* if $G = G^{(k)}$. In this talk we discuss k-closures of nilpotent groups.

Theorem. If G is a finite nilpotent permutation group, and $k \geq 2$, then $G^{(k)}$ is the direct product of k-closures of Sylow subgroups of G.

This theorem generalizes results of [2,3] and provides a criterion of the k-closedness for finite nilpotent permutation groups.

Corollary. For $k \geq 2$, a finite nilpotent permutation group G is k-closed if and only if every Sylow subgroup of G is k-closed.

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References

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