k-closures of finite nilpotent permutation groups

Dmitry Churikov
Sobolev Institute of Mathematics, Novosibirsk, Russia
Novosibirsk State University, Novosibirsk, Russia
churikovdv@gmail.com

Let G be a permutation group on a finite set Ω. Denote the set of orbits of the componentwise action of G on Ω^k by $\mathrm{Orb}(G, \Omega^k)$. Wielandt [1] defined the *k-closure* of G to be the group

$$G^{(k)} = \mathrm{Aut}(\mathrm{Orb}(G, \Omega^k)) = \{g \in \mathrm{Sym}(\Omega) \mid O^g = O \ \forall O \in \mathrm{Orb}(G, \Omega^k)\}.$$

A permutation group is called *k-closed* if $G = G^{(k)}$. In this talk we discuss k-closures of nilpotent groups.

Theorem. *If G is a finite nilpotent permutation group, and $k \geq 2$, then $G^{(k)}$ is the direct product of k-closures of Sylow subgroups of G.*

This theorem generalizes results of [2,3] and provides a criterion of the k-closedness for finite nilpotent permutation groups.

Corollary. *For $k \geq 2$, a finite nilpotent permutation group G is k-closed if and only if every Sylow subgroup of G is k-closed.*

Acknowledgments. The work is supported by the Mathematical Center in Akademgorodok under the agreement No. 075-15-2019-1613 with the Ministry of Science and Higher Education of the Russian Federation.

References

[1] H. W. Wielandt, *Permutation groups through invariant relations and invariant functions*, Lecture Notes, Ohio State University, Ohio (1969).

[2] D. Churikov, C. Praeger, Finite totally k-closed groups, *Trudy Instituta Matematiki i Mekhaniki UrO RAN*, **1** (2021) 240–245.

[3] D. Churikov, I. Ponomarenko, On 2-closed abelian permutation groups, arXiv:2011.12011 (2020).