On maximal cliques in Paley graphs of square order

Sergey Goryainov
Chelyabinsk State University, Chelyabinsk, Russia
sergey.goryainov3@gmail.com

This is joint work with Alexander Masley and Leonid Shalaginov

In [1], Blokhuis studied maximum cliques in Paley graphs of square order $P(q^2)$. It was shown that a clique of size q in $P(q^2)$ is necessarily a quadratic line in the corresponding affine plane $A(2,q)$.

Let $r(q)$ denote the reminder after division of q by 4. In [2], for any odd prime power q, a maximal (but not maximum) clique in $P(q^2)$ of size $\frac{q+r(q)}{2}$ was constructed.

In [3], for any odd prime power q, a maximal clique in $P(q^2)$ of the same size $\frac{q+r(q)}{2}$ was constructed. This clique was shown to have a remarkable connection with eigenfunctions of $P(q^2)$ that have minimum cardinality of support $q+1$.

In this talk, we discuss the constructions of maximal cliques from [2] and [3] and establish a correspondence between them.

Acknowledgments. Sergey Goryainov and Leonid Shalaginov are supported by RFBR according to the research project 20-51-53023.

References

[1] A. Blokhuis, On subsets of $GF(q^2)$ with square differences. *Indag. Math.* **46** (1984) 369–372.

[2] R. D. Baker, G. L. Ebert, J. Hemmeter, A. J. Woldar, Maximal cliques in the Paley graph of square order. *J. Statist. Plann. Inference* **56** (1996) 33–38.

[3] S. V. Goryainov, V. V. Kabanov, L. V. Shalaginov, A. A. Valyuzhenich, On eigenfunctions and maximal cliques of Paley graphs of square order. *Finite Fields and Their Applications* **52** (2018) 361–369.

[4] S. V. Goryainov, A. V. Masley, L. V. Shalaginov, On a correspondence between maximal cliques in Paley graphs of square order. arXiv:2102.03822 (2021).