Contribution ID: **8** Type: **not specified**

On coincidence of Gruenberg–Kegel graphs of non-isomorphic finite groups

We use mainly standard notation and terminology (see [1]). Let G be a finite group. Denote by $\pi(G)$ the set of all prime divisors of the order of G and by $\omega(G)$ the *spectrum* of G, i.e. the set of all its element orders. The set $\omega(G)$ defines the *Gruenberg–Kegel* graph (or the *prime graph*) $\Gamma(G)$ of G; in this simple graph the vertex set is $\pi(G)$, and distinct vertices p and q are adjacent if and only if $pq \in \omega(G)$.

The problem of description of the cases when Gruenberg–Kegel graphs of non-isomorphic finite groups coincide naturally arises. One of the most interesting cases of this problem is when Gruenberg–Kegel graphs of non-isomorphic finite groups are both disconnected and coincide.

Recall some known definitions. A finite group G is a *Frobenius group* if there is a non-trivial subgroup C of G such that $C \cap gCg^{-1} = \{1\}$ whenever $g \notin C$. A subgroup C is a *Frobenius complement* of G. Let $K = \{1\} \cup (G \setminus \bigcup_{g \in G} gCg^{-1})$. Then K is a normal subgroup of a Frobenius group G with a Frobenius complement C which is called the *Frobenius kernel* of G. A finite group G is a 2-Frobenius group if $G = ABC$, where A and AB are normal subgroups of G, AB and BC are Frobenius groups with kernels A and B and complements B and C, respectively. It is known that each 2-Frobenius group is solvable. The *socle* $Soc(G)$ of a finite group G is the subgroup of G generated by the set of all its non-trivial minimal normal subgroups. A finite group G is *almost simple* if $Soc(G)$ is a finite nonabelian simple group.

By the Gruenberg–Kegel Theorem, if G is a finite group with disconnected Gruenberg–Kegel graph, then either G is a Frobenius group or either G is a 2-Frobenius group or G is an extension of a nilpotent group by an almost simple group. In [2] all the cases of coincidence of Gruenberg–Kegel graphs of a finite simple group and of a Frobenius or a 2-Frobenius group were described. Moreover, we can obtain the complete list of almost simple groups whose Gruenberg–Kegel graphs coincide with Gruenberg–Kegel graphs of solvable Frobenius groups or 2-Frobenius groups directly from the main results of the papers [3] and [4].

In this talk we concentrate on a list of almost simple (but not simple) groups whose Gruenberg–Kegel graphs coincide with Gruenberg–Kegel graphs of non-solvable Frobenius groups. We prove the following theorem.

Theorem 1. *Let G be an almost simple but not simple group. If $\Gamma(G) = \Gamma(H)$, where H is a non-solvable Frobenius group, then either G is one of the groups $Aut(M_{12})$, S_7, $Aut(^2F_4(q))$, $Aut(PSU_4(2))$, $L_3(4).2_3$, $PSL_4(3).2_2$, $PSL_4(3).2_3$, $PSL_2(11).2 \cong PGL_2(11)$, $PSL_2(19).2 \cong PGL_2(19)$, $PGL_2(49).2_1 \cong PGL_2(49)$, $PSL_2(25).2_2$, $PSL_2(81).2_1$, $PSL_2(81).4_1$, $PSL_2(81).4_2$; or $Soc(G) \in \{PSL_3(q), PSU_3(q)\}$.*

Acknowledgments. The work has been supported by the Russian Science Foundation (project 19-71-10067). This is joint work with Natalia Maslova.

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Primary author: ILENKO, Kristina

Co-author: MASLOVA, Natalia (Krasovskii Institute of Mathematics and Mechanics UB RAS)

Presenter: ILENKO, Kristina

Track Classification: Oral presentation