The nonexistence of distance-regular graphs with intersection array $\{56, 42, 20; 1, 6, 28\}$ in which some local subgraph is the 7×8-grid.

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There is an infinite sequence of formally self-dual classical distance-regular graphs Γ with classical parameters $b = 2$, $\alpha = 1$, $\beta = n - 1$, $v = n^3$ $(n > 5)$ (see, for example, [1]). If n is a power of 2, then there exists a distance-regular graph Γ with intersection array $\{7(n-1), 6(n-2), 4(n-4); 1, 6, 28\}$ and each distance-regular graph with these parameters is a bilinear forms graph. Moreover, all these graphs are locally grid (see [2]).

We consider graphs Γ with intersection array $\{7(n-1), 6(n-2), 4(n-4); 1, 6, 28\}$, where n is not a power of 2. By the results of Metsch [3,6], if $n \geq 71$, then graphs with intersection array $\{7(n-1), 6(n-2), 4(n-4); 1, 6, 28\}$ don't exist.

If $n = 6$, then the intersection array of Γ is $\{35, 24, 8; 1, 6, 28\}$. The nonexistence of a graph with this intersection array was proved by A. Jurishich and J. Vidali [4]. The case when $n = 7$ was ruled out by I.N. Belousov and A.A. Makhnev [5]. The proof was based on counting of some triple intersection numbers.

In this work, we consider the case $n = 9$. In this case, there are 6 admissible spectra for the integral local subgraph. One of them is relate to the 7×8-grid.

We prove that a distance-regular graph with intersection array $\{56, 42, 20; 1, 6, 28\}$ does not exist, if some its local subgraph is the 7×8-grid.

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