On Cayley isomorphism property for abelian groups

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A finite group G is called a *DCI-group* if every two isomorphic Cayley digraphs over G are Cayley isomorphic, i.e. there exists an isomorphism between these digraphs that is also an automorphism of G. One of the motivations to study DCI-groups comes from the Cayley graph isomorphism problem. Suppose that G is a DCI-group. Then to determine whether two Cayley digraphs $\mathrm{Cay}(G, S)$ and $\mathrm{Cay}(G, T)$ are isomorphic, we only need to check the existence of $\varphi \in \mathrm{Aut}(G)$ with $S^{\varphi} = T$. The latter, usually, is much easier.

The definition of a DCI-group goes back to Ádám who conjectured [1], in our terms, that every cyclic group is DCI. This conjecture was proved to be false. The problem of determining all finite DCI-groups was raised by Babai and Frankl [2]. One of the crucial steps towards the classification of all DCI-groups is to determine abelian DCI-groups. It was proved that every abelian DCI-group is the direct product of groups of coprime orders each of which is elementary abelian or isomorphic to $\mathbb{Z}_4$ (see [3, Theorem 8.8]). However, the classification of abelian DCI-groups is far from complete. In the talk we discuss on new infinite families of abelian DCI-groups and approaches to determining whether a given group is DCI.

References

[1] *A. Ádám*, Research Problem 2-10, J. Combin. Theory, **2** (1967) 393.

[2] *L. Babai, P. Frankl*, Isomorphisms of Cayley graphs I, Colloq. Math. Soc. János Bolyai, **18**, North-Holland, Amsterdam (1978) 35–52.

[3] *C. H. Li*, On isomorphisms of finite Cayley graphs – a survey, Discrete Math., **256**, Nos. 1-2 (2002) 301–334.