Eigenfunctions of the Star graphs for all non-zero eigenvalues

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Let G be a finite group and S be a subset of G which does not contain the identity element and is closed under inversion. The Cayley graph $\mathrm{Cay}(G, S)$ is a graph with the vertex set G in which two vertices x, y are adjacent if and only if $xy^{-1} \in S$. For $\Omega = \{1, \dots, n\}$, $n \geqslant 2$, we consider the symmetric group Sym_Ω and put $S = \{(1\ i) \mid i \in \{2, \dots, n\}\}$. The *Star graph* $S_n = \mathrm{Cay}(\mathrm{Sym}_\Omega, S)$ is the Cayley graph over the symmetric group Sym_Ω with the generating set S.

A function $f : V(\Gamma) \to \mathbb{R}$ is called an *eigenfunction* of a graph Γ corresponding to an eigenvalue θ if $f \not\equiv 0$ and the equality

$$\theta \cdot f(x) = \sum_{y \in N(x)} f(y) \tag{1}$$

holds for any its vertex x, where $N(x)$ is the neighborhood of x in Γ.

The Star graph S_n, $n \geq 2$, is known to be integral (see [2]), and its spectrum consists of all integers in the range from $-(n-1)$ to $n-1$ (except 0 when $n = 2, 3$). Despite of the fact that spectral properties of the Star graph were studied (see [1–3, 5]), no explicit construction for the eigenfunctions was known.

In [4], an explicit construction of eigenfunctions of S_n, $n \geq 3$, for all eigenvalues θ with $\frac{n-2}{2} < \theta < n-1$ was presented.

In this work, we generalize ideas from [4] and present eigenfunctions of the Star graph S_n, $n \geq 3$, for all its non-zero eigenvalues.

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References

[1] A. Abdollahi, E. Vatandoost, Which Cayley graphs are integral? *The Electronic Journal of Combinatorics*, **16** (2009) 6–7.

[2] G. Chapuy, V. Feray, A note on a Cayley graph of Sym_n, *arXiv:1202.4976v2* (2012) 1–3.

[3] J. Friedman, On Cayley graphs on the symmetric group generated by transpositions, *Combinatorica* **20**(4) (2000) 505–519.

[4] S. Goryainov, V. V. Kabanov, E. Konstantinova, L. Shalaginov, A. Valyuzhenich, PI-eigenfunctions of the Star graphs, *Linear Algebra and its Applications*, **586** (2020) 7–27.

[5] R. Krakovski, B. Mohar, Spectrum of Cayley graphs on the symmetric group generated by transposition, *Linear Algebra and its Applications*, **437** (2012) 1033–1039.