The Awesome Diversity of the Clique Graph Operator Dynamics

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Abstract

The class of all graphs $\mathcal{G}$ together with an operator $\Phi : \mathcal{G} \to \mathcal{G}$ constitute a *graph dynamic system*. Special interest is paid to the effects of applying the respective *iterated operators*: $\Phi^0(G) = G$, $\Phi^{n+1}(G) = \Phi(\Phi^n(G))$. Many natural questions arise in this context including, which graphs are Φ-*invariant* ($\Phi(G) \cong G$), Φ-*convergent* ($\Phi^n(G) \cong \Phi^m(G)$ for some $n < m$) or Φ-*divergent* ($\lim_{n\to\infty} |\Phi^n(G)| = \infty$).

In particular, the clique operator K transforms a graph G into the clique graph $K(G)$, which is the intersection graph of the maximal cliques of G. The clique graph operator is one of the most studied due to the huge richness of the corresponding graph dynamic system. Clique graphs have even been used in Loop Quantum Gravity to explain how the quantum spacetime foam (and classic spacetime) could emerge from a more basic, discrete reality underlying at the Plank's length scale.

More recently, the problem of algorithmic decidability of the K-divergence has begun to be studied. For this, some electronic gates and circuits have been simulated within the clique graph dynamics. And the preliminary results seem to suggest that the clique graph dynamics has at least the computational power of Linearly Bounded Automata (i.e. computers with finite memory like real computers, but unlike Turing machines).

In this talk, we shall show a very visual general overview of the several breakthroughs made in the last 50 years.

This talk is based on several works by C. Cedillo, F. Escalante, F. Larrión, V. Neumann-Lara, M. Pizaña, J. Szwarcfiter, R. Villarroel-Flores and others.